An approach to selection theory for dendritic growth enabling the treatment of general bulk equations

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Abstract

Asymptotic matching in the complex plane is a strategy for calculating exponentially small terms that has been pioneered for nonlinear equations by Kruskal and Segur. The method has been successfully applied to pattern-forming systems that could be cast into the form of a single ordinary differential or differential-integral equation. Examples are viscous fingering, dendritic crystal growth, or capillary water waves. Interesting problems that are modeled by nonlinear field equations with free boundaries, however, remained intractable.

We show how to combine asymptotic matching in the complex plane with Zauderer's decomposition scheme of nonlinear partial differential equations to study this class of problems. The method is exemplified by dendritic growth limited by nonlinear heat transport.

The whole program is based on a free boundary presentation of the model problems. We first reproduce and extend the results previously obtained for linear heat transport from differential-integral equations. The derivation is thereby considerably simplified. We also consider the modifications due to a thermal Kapitza resistance. The nonlinear transport equations considered model diffusive heat transport accompanied by an inviscid fluid flow.

Exponentially small terms behave frequently counter-intuitive. We develop a graphical method to ease intuition for the order of magnitude of the various effects. It turns out that a Kapitza resistance has a surprisingly large effect.