

# THE MATHSCOPE

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translated by Phạm Văn Thuận, Eckard Specht

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## Vol I, Problems in Mathematics Journal for the Youth

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The Mathscape is a free problem resource selected from mathematical problem solving journals in Vietnam. This freely accessible collection is our effort to introduce elementary mathematics problems to foreign friends for either recreational or professional use. We would like to give you a new taste of Vietnamese mathematical culture. Whatever the purpose, we welcome suggestions and comments from you all. More communications can be addressed to PHẠM VĂN THUẬN of Hanoi University, at [pvthuan@vnu.edu.vn](mailto:pvthuan@vnu.edu.vn)

It's now not too hard to find problems and solutions on the Internet due to the increasing number of websites devoted to mathematical problem solving. It is our hope that this collection saves you considerable time searching the problems you really want. We intend to give an outline of solutions to the problems in the future. Now enjoy these "cakes" from Vietnam first.

Pham Van Thuan

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**153. 1 (Nguyễn Đông Yên)** Prove that if  $y \geq y^3 + x^2 + |x| + 1$ , then  $x^2 + y^2 \geq 1$ . Find all pairs of  $(x, y)$  such that the first inequality holds while equality in the second one attains.

**153. 2 (Tạ Văn Tự)** Given natural numbers  $m, n$ , and a real number  $a > 1$ , prove the inequality

$$a^{\frac{2n}{m}} - 1 \geq n(a^{\frac{n+1}{m}} - a^{\frac{n-1}{m}}).$$

**153. 3 (Nguyễn Minh Đức)** Prove that for each  $0 < \epsilon < 1$ , there exists a natural number  $n_0$  such that the coefficients of the polynomial

$$(x + y)^n(x^2 - (2 - \epsilon)xy + y^2)$$

are all positive for each natural number  $n \geq n_0$ .

**235. 1 (Đặng Hùng Thắng)** Given real numbers  $x, y, z$  such that

$$\begin{aligned} a + b &= 6, \\ ax + by &= 10, \\ ax^2 + by^2 &= 24, \\ ax^3 + by^3 &= 62, \end{aligned}$$

determine  $ax^4 + by^4$ .

**235. 2 (Hà Đức Vương)** Let  $ABC$  be a triangle, let  $D$  be a fixed point on the opposite ray of ray  $BC$ . A variable ray  $D_x$  intersects the sides  $AB, AC$  at  $E, F$ , respectively. Let  $M$  and  $N$  be the midpoints of  $BF, CE$ , respectively. Prove that the line  $MN$  has a fixed point.

**235. 3 (Đàm Văn Nhĩ)** Find the maximum value of

$$\frac{a}{bcd + 1} + \frac{b}{cda + 1} + \frac{c}{dab + 1} + \frac{d}{abc + 1},$$

where  $a, b, c, d \in [0, 1]$ .

**235. 4 (Trần Nam Dũng)** Let  $M$  be any point in the plane of an equilateral triangle  $ABC$ . Denote by  $x, y, z$  the distances from  $P$  to the vertices and  $p, q, r$  the distances from  $M$  to the sides of the triangle. Prove that

$$p^2 + q^2 + r^2 \geq \frac{1}{4}(x^2 + y^2 + z^2),$$

and that this inequality characterizes all equilateral triangles in the sense that we can always choose a point  $M$  in the plane of a non-equilateral triangle such that the inequality is not true.

**241.1 (Trần Nam Dũng)** Given  $n$  real numbers  $x_1, x_2, \dots, x_n$  in the interval  $[0, 1]$ , prove that

$$\left\lfloor \frac{n}{2} \right\rfloor \geq x_1(1-x_2) + x_2(1-x_3) + \dots + x_{n-1}(1-x_n) + x_n(1-x_1).$$

**241.2 (Trần Xuân Đáng)** Prove that in any acute triangle  $ABC$

$$\sin A \sin B + \sin B \sin C + \sin C \sin A \geq (1 + \sqrt{2 \cos A \cos B \cos C})^2.$$

**243.1 (Huỳnh Minh Việt)** Given real numbers  $x, y, z$  such that  $x^2 + y^2 + z^2 = k$ ,  $k > 0$ , prove the inequality

$$\frac{2}{k}xyz - \sqrt{2k} \leq x + y + z \leq \frac{2}{k}xyz + \sqrt{2k}.$$

**244.1 (Thái Viết Bảo)** Given a triangle  $ABC$ , let  $D$  and  $E$  be points on the sides  $AB$  and  $AC$ , respectively. Points  $M, N$  are chosen on the line segment  $DE$  such that  $DM = MN = NE$ . Let  $BC$  intersect the rays  $AM$  and  $AN$  at  $P$  and  $Q$ , respectively. Prove that if  $BP < PQ$ , then  $PQ < QC$ .

**244.2 (Ngô Văn Thái)** Prove that if  $0 < a, b, c \leq 1$ , then

$$\frac{1}{a+b+c} \geq \frac{1}{3} + (1-a)(1-b)(1-c).$$

**244.3 (Trần Chí Hòa)** Given three positive real numbers  $x, y, z$  such that  $xy + yz + zx + \frac{2}{a}xyz = a^2$ , where  $a$  is a given positive number, find the maximum value of  $c(a)$  such that the inequality  $x + y + z \geq c(a)(xy + yz + zx)$  holds.

**244.4 (Đàm Văn Nhĩ)** The sequence  $\{p(n)\}$  is recursively defined by

$$p(1) = 1, \quad p(n) = 1p(n-1) + 2p(n-2) + \dots + (n-1)p(n-1)$$

for  $n \geq 2$ . Determine an explicit formula for  $n \in \mathbb{N}^*$ .

**244.5 (Nguyễn Vũ Lương)** Solve the system of equations

$$\begin{aligned} 4xy + 4(x^2 + y^2) + \frac{3}{(x+y)^2} &= \frac{85}{3}, \\ 2x + \frac{1}{x+y} &= \frac{13}{3}. \end{aligned}$$

**248.1 (Trần Văn Vương)** Given three real numbers  $x, y, z$  such that  $x \geq 4$ ,  $y \geq 5$ ,  $z \geq 6$  and  $x^2 + y^2 + z^2 \geq 90$ , prove that  $x + y + z \geq 16$ .

**248. 2 (Đỗ Thanh Hân)** Solve the system of equations

$$x^3 - 6z^2 + 12z - 8 = 0,$$

$$y^3 - 6x^2 + 12x - 8 = 0,$$

$$z^3 - 6y^2 + 12y - 8 = 0.$$

**248. 3 (Phương Tố Tử)** Let the incircle of an equilateral triangle  $ABC$  touch the sides  $AB, AC, BC$  respectively at  $C', B'$  and  $A'$ . Let  $M$  be any point on the minor arc  $B'C'$ , and  $H, K, L$  the orthogonal projections of  $M$  onto the sides  $BC, AC$  and  $AB$ , respectively. Prove that

$$\sqrt{MH} = \sqrt{MK} + \sqrt{ML}.$$

**250. 1 (Đặng Hùng Thắng)** Find all pairs  $(x, y)$  of natural numbers  $x > 1, y > 1$ , such that  $3x + 1$  is divisible by  $y$  and simultaneously  $3y + 1$  is divisible by  $x$ .

**250. 2 (Nguyễn Ngọc Khoa)** Prove that there exists a polynomial with integer coefficients such that its value at each root  $t$  of the equation  $t^8 - 4t^4 + 1 = 0$  is equal to the value of

$$f(t) = \frac{5t^2}{t^8 + t^5 - t^3 - 5t^2 - 4t + 1}$$

for this value of  $t$ .

**250. 3 (Nguyễn Khắc Minh)** Consider the equation  $f(x) = ax^2 + bx + c$  where  $a < b$  and  $f(x) \geq 0$  for all real  $x$ . Find the smallest possible value of

$$p = \frac{a + b + c}{b - a}.$$

**250. 4 (Trần Đức Thịnh)** Given two fixed points  $B$  and  $C$ , let  $A$  be a variable point on the semiplanes with boundary  $BC$  such that  $A, B, C$  are not collinear. Points  $D, E$  are chosen in the plane such that triangles  $ADB$  and  $AEC$  are right isosceles and  $AD = DB, EA = EC$ , and  $D, C$  are on different sides of  $AB$ ;  $B, E$  are on different sides of  $AC$ . Let  $M$  be the midpoint of  $DE$ , prove that line  $AM$  has a fixed point.

**250. 5 (Trần Nam Dũng)** Prove that if  $a, b, c > 0$  then

$$\frac{1}{2} + \frac{a^2 + b^2 + c^2}{ab + bc + ca} \geq \frac{a}{b + c} + \frac{b}{c + a} + \frac{c}{a + b} \geq \frac{1}{2} \left( 4 - \frac{ab + bc + ca}{a^2 + b^2 + c^2} \right).$$

**250. 6 (Phạm Ngọc Quang)** Given a positive integer  $m$ , show that there exist prime integers  $a, b$  such that the following conditions are simultaneously satisfied:

$$|a| \leq m, \quad |b| \leq m \quad \text{and} \quad 0 < a + b\sqrt{2} \leq \frac{1 + \sqrt{2}}{m + 2}.$$

**250. 7 (Lê Quốc Hán)** Given a triangle  $ABC$  such that  $\cot A, \cot B$  and  $\cot C$  are respectively terms of an arithmetic progression. Prove that  $\angle GAC = \angle GBA$ , where  $G$  is the centroid of the triangle.

**250. 8 (Nguyễn Minh Đức)** Find all polynomials with real coefficients  $f(x)$  such that  $\cos(f(x)), x \in \mathbb{R}$ , is a periodic function.

**251. 1 (Nguyễn Duy Liên)** Find the smallest possible natural number  $n$  such that  $n^2 + n + 1$  can be written as a product of four prime numbers.

**251. 2 (Nguyễn Thanh Hải)** Given a cubic equation

$$x^3 - px^2 + qx - p = 0,$$

where  $p, q \in \mathbb{R}^*$ , prove that if the equation has only real roots, then the inequality

$$p \geq \left(\frac{1}{4} + \frac{\sqrt{2}}{8}\right)(q + 3)$$

holds.

**251. 3 (Nguyễn Ngọc Bình Phương)** Given a circle with center  $O$  and radius  $r$  inscribed in triangle  $ABC$ . The line joining  $O$  and the midpoint of side  $BC$  intersects the altitude from vertex  $A$  at  $I$ . Prove that  $AI = r$ .

**258. 1 (Đặng Hùng Thắng)** Let  $a, b, c$  be positive integers such that

$$a^2 + b^2 = c^2(1 + ab),$$

prove that  $a \geq c$  and  $b \geq c$ .

**258. 2 (Nguyễn Việt Hải)** Let  $D$  be any point between points  $A$  and  $B$ . A circle  $\Gamma$  is tangent to the line segment  $AB$  at  $D$ . From  $A$  and  $B$ , two tangents to the circle are drawn, let  $E$  and  $F$  be the points of tangency, respectively,  $D$  distinct from  $E, F$ . Point  $M$  is the reflection of  $A$  across  $E$ , point  $N$  is the reflection of  $B$  across  $F$ . Let  $EF$  intersect  $AN$  at  $K$ ,  $BM$  at  $H$ . Prove that triangle  $DKH$  is isosceles, and determine the center of  $\Gamma$  such that  $\triangle DKH$  is equilateral.

**258.3 (Vi Quốc Dũng)** Let  $AC$  be a fixed line segment with midpoint  $K$ , two variable points  $B, D$  are chosen on the line segment  $AC$  such that  $K$  is the midpoint of  $BD$ . The bisector of angle  $\angle BCD$  meets lines  $AB$  and  $AD$  at  $I$  and  $J$ , respectively. Suppose that  $M$  is the second intersection of circumcircle of triangle  $ABD$  and  $AIJ$ . Prove that  $M$  lies on a fixed circle.

**258.4 (Đặng Kỳ Phong)** Find all functions  $f(x)$  that satisfy simultaneously the following conditions

- i)  $f(x)$  is defined and continuous on  $\mathbb{R}$ ;
- ii) for each set of 1997 numbers  $x_1, x_2, \dots, x_{1997}$  such that  $x_1 < x_2 < \dots < x_n$ , the inequality

$$f(x_{999}) \geq \frac{1}{1996} (f(x_1) + f(x_2) + \dots + f(x_{998}) + f(x_{1000}) + f(x_{1001}) + \dots + f(x_{1997})).$$

holds.

**259.1 (Nguyễn Phước)** Solve the equation

$$(x + 3\sqrt{x} + 2)(x + 9\sqrt{x} + 18) = 168x.$$

**259.2 (Viên Ngọc Quang)** Given four positive real numbers  $a, b, c$  and  $d$  such that the quartic equation  $ax^4 - ax^3 + bx^2 - cx + d = 0$  has four roots in the interval  $(0, \frac{1}{2})$ , the roots not being necessarily distinct. Prove that

$$21a + 164c \geq 80b + 320d.$$

**259.3 (Hồ Quang Vinh)** Given is a triangle  $ABC$ . The excircle of  $ABC$  inside angle  $A$  touches side  $BC$  at  $A_1$ , and the other two excircles inside angles  $B, C$  touch sides  $CA$  and  $AB$  at  $B_1, C_1$ , respectively. The lines  $AA_1, BB_1, CC_1$  are concurrent at point  $N$ . Let  $D, E, F$  be the orthogonal projections of  $N$  onto the sides  $BC, CA$  and  $AB$ , respectively. Suppose that  $R$  is the circumradius and  $r$  the inradius of triangle  $ABC$ . Denote by  $S(XYZ)$  the area of triangle  $XYZ$ , prove that

$$\frac{S(DEF)}{S(ABC)} = \frac{r}{R} \left(1 - \frac{r}{R}\right).$$

**261.1 (Hồ Quang Vinh)** Given a triangle  $ABC$ , its internal angle bisectors  $BE$  and  $CF$ , and let  $M$  be any point on the line segment  $EF$ . Denote by  $S_A, S_B$ , and  $S_C$  the areas of triangles  $MBC, MCA$ , and  $MAB$ , respectively. Prove that

$$\frac{\sqrt{S_B} + \sqrt{S_C}}{\sqrt{S_A}} \leq \sqrt{\frac{AC + AB}{BC}},$$

and determine when equality holds.

**261. 2 (Editorial Board)** Find the maximum value of the expression

$$A = 13\sqrt{x^2 - x^4} + 9\sqrt{x^2 + x^4} \quad \text{for } 0 \leq x \leq 1.$$

**261. 3 (Editorial Board)** The sequence  $(a_n)$ ,  $n = 1, 2, 3, \dots$ , is defined by  $a_1 > 0$ , and  $a_{n+1} = ca_n^2 + a_n$  for  $n = 1, 2, 3, \dots$ , where  $c$  is a constant. Prove that

$$\text{a) } a_n \geq \sqrt{c^{n-1}n^n a_1^{n+1}}, \quad \text{and}$$

$$\text{b) } a_1 + a_2 + \dots + a_n > n \left( na_1 - \frac{1}{c} \right) \quad \text{for } n \in \mathbb{N}.$$

**261. 4 (Editorial Board)** Let  $X, Y, Z$  be the reflections of  $A, B$ , and  $C$  across the lines  $BC, CA$ , and  $AB$ , respectively. Prove that  $X, Y$ , and  $Z$  are collinear if and only if

$$\cos A \cos B \cos C = -\frac{3}{8}.$$

**261. 5 (Vinh Competition)** Prove that if  $x, y, z > 0$  and  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$  then the following inequality holds:

$$\left(1 - \frac{1}{1+x^2}\right) \left(1 - \frac{1}{1+y^2}\right) \left(1 - \frac{1}{1+z^2}\right) > \frac{1}{2}.$$

**261. 6 (Đỗ Văn Đức)** Given four real numbers  $x_1, x_2, x_3, x_4$  such that  $x_1 + x_2 + x_3 + x_4 = 0$  and  $|x_1| + |x_2| + |x_3| + |x_4| = 1$ , find the maximum value of  $\prod_{1 \leq i < j \leq 4} (x_i - x_j)$ .

**261. 7 (Đoàn Quang Mạnh)** Given a rational number  $x \geq 1$  such that there exists a sequence of integers  $(a_n)$ ,  $n = 0, 1, 2, \dots$ , and a constant  $c \neq 0$  such that  $\lim_{n \rightarrow \infty} (cx^n - a_n) = 0$ . Prove that  $x$  is an integer.

**262. 1 (Ngô Văn Hiệp)** Let  $ABC$  an equilateral triangle of side length  $a$ . For each point  $M$  in the interior of the triangle, choose points  $D, E, F$  on the sides  $CA, AB$ , and  $BC$ , respectively, such that  $DE = MA, EF = MB$ , and  $FD = MC$ . Determine  $M$  such that  $\triangle DEF$  has smallest possible area and calculate this area in terms of  $a$ .

**262. 2 (Nguyễn Xuân Hùng)** Given is an acute triangle with altitude  $AH$ . Let  $D$  be any point on the line segment  $AH$  not coinciding with the endpoints of this segment and the orthocenter of triangle  $ABC$ . Let ray  $BD$  intersect  $AC$  at  $M$ , ray  $CD$  meet  $AB$  at  $N$ . The line perpendicular to  $BM$  at  $M$  meets the line perpendicular to  $CN$  at  $N$  in the point  $S$ . Prove that  $\triangle ABC$  is isosceles with base  $BC$  if and only if  $S$  is on line  $AH$ .

**262. 3 (Nguyễn Duy Liên)** The sequence  $(a_n)$  is defined by

$$a_0 = 2, \quad a_{n+1} = 4a_n + \sqrt{15a_n^2 - 60} \quad \text{for } n \in \mathbb{N}.$$

Find the general term  $a_n$ . Prove that  $\frac{1}{5}(a_{2n} + 8)$  can be expressed as the sum of squares of three consecutive integers for  $n \geq 1$ .

**262. 4 (Tuấn Anh)** Let  $p$  be a prime,  $n$  and  $k$  positive integers with  $k > 1$ . Suppose that  $b_i, i = 1, 2, \dots, k$ , are integers such that

i)  $0 \leq b_i \leq k - 1$  for all  $i$ ,

ii)  $p^{nk-1}$  is a divisor of  $(\sum_{i=1}^k p^{nb_i}) - p^{n(k-1)} - p^{n(k-2)} - \dots - p^n - 1$ .

Prove that the sequence  $(b_1, b_2, \dots, b_k)$  is a permutation of the sequence  $(0, 1, \dots, k - 1)$ .

**262. 5 (Đoàn Thế Phiệt)** Without use of any calculator, determine

$$\sin \frac{\pi}{14} + 6 \sin^2 \frac{\pi}{14} - 8 \sin^4 \frac{\pi}{14}.$$

**264. 1 (Trần Duy Hình)** Prove that the sum of all squares of the divisors of a natural number  $n$  is less than  $n^2\sqrt{n}$ .

**264. 2 (Hoàng Ngọc Cảnh)** Given two polynomials

$$f(x) = x^4 - (1 + e^x) + e^2, \quad g(x) = x^4 - 1,$$

prove that for distinct positive numbers  $a, b$  satisfying  $a^b = b^a$ , we have  $f(a)f(b) < 0$  and  $g(a)g(b) > 0$ .

**264. 3 (Nguyễn Phú Yên)** Solve the equation

$$\frac{(x-1)^4}{(x^2-3)^2} + (x^2-3)^4 + \frac{1}{(x-1)^2} = 3x^2 - 2x - 5.$$

**264. 4 (Nguyễn Minh Phương, Nguyễn Xuân Hùng)** Let  $I$  be the in-center of triangle  $ABC$ . Rays  $AI$ ,  $BI$ , and  $CI$  meet the circumcircle of triangle  $ABC$  again at  $X$ ,  $Y$ , and  $Z$ , respectively. Prove that

a)  $IX + IY + IZ \geq IA + IB + IC$ ,      b)  $\frac{1}{IX} + \frac{1}{IY} + \frac{1}{IZ} \geq \frac{3}{R}$ .

**265. 1 (Vũ Đình Hòa)** The lengths of the four sides of a convex quadrilateral are natural numbers such that the sum of any three of them is divisible by the fourth number. Prove that the quadrilateral has two equal sides.

**265. 2 (Đàm Văn Nhi)** Let  $AD$ ,  $BE$ , and  $CF$  be the internal angle bisectors of triangle  $ABC$ . Prove that  $p(DEF) \leq \frac{1}{2}p(ABC)$ , where  $p(XYZ)$  denotes the perimeter of triangle  $XYZ$ . When does equality hold?

**266. 1 (Lê Quang Nãm)** Given real numbers  $x, y, z \geq -1$  satisfying  $x^3 + y^3 + z^3 \geq x^2 + y^2 + z^2$ , prove that  $x^5 + y^5 + z^5 \geq x^2 + y^2 + z^2$ .

**266. 2 (Đặng Nhơn)** Let  $ABCD$  be a rhombus with  $\angle A = 120^\circ$ . A ray  $Ax$  and  $AB$  make an angle of  $15^\circ$ , and  $Ax$  meets  $BC$  and  $CD$  at  $M$  and  $N$ , respectively. Prove that

$$\frac{3}{AM^2} + \frac{3}{AN^2} = \frac{4}{AB^2}.$$

**266. 3 (Hà Duy Hưng)** Given an isosceles triangle with  $\angle A = 90^\circ$ . Let  $M$  be a variable point on line  $BC$ , ( $M$  distinct from  $B, C$ ). Let  $H$  and  $K$  be the orthogonal projections of  $M$  onto lines  $AB$  and  $AC$ , respectively. Suppose that  $I$  is the intersection of lines  $CH$  and  $BK$ . Prove that the line  $MI$  has a fixed point.

**266. 4 (Lưu Xuân Tình)** Let  $x, y$  be real numbers in the interval  $(0, 1)$  and  $x + y = 1$ , find the minimum of the expression  $x^x + y^y$ .

**267. 1 (Đỗ Thanh Hân)** Let  $x, y, z$  be real numbers such that

$$\begin{aligned}x^2 + z^2 &= 1, \\y^2 + 2y(x + z) &= 6.\end{aligned}$$

Prove that  $y(z - x) \leq 4$ , and determine when equality holds.

**267. 2 (Lê Quốc Hán)** In triangle  $ABC$ , medians  $AM$  and  $CN$  meet at  $G$ . Prove that the quadrilateral  $BMGN$  has an incircle if and only if triangle  $ABC$  is isosceles at  $B$ .

**267. 3 (Trần Nam Dũng)** In triangle  $ABC$ , denote by  $a, b, c$  the side lengths, and  $F$  the area. Prove that

$$F \leq \frac{1}{16}(3a^2 + 2b^2 + 2c^2),$$

and determine when equality holds. Can we find another set of the coefficients of  $a^2$ ,  $b^2$ , and  $c^2$  for which equality holds?

**268. 1 (Đỗ Kim Sơn)** In a triangle, denote by  $a, b, c$  the side lengths, and let  $r, R$  be the inradius and circumradius, respectively. Prove that

$$a(b + c - a)^2 + b(c + a - b)^2 + c(a + b - c)^2 \leq 6\sqrt{3}R^2(2R - r).$$

**268. 2 (Đặng Hùng Thắng)** The sequence  $(a_n)$ ,  $n \in \mathbb{N}$ , is defined by

$$a_0 = a, \quad a_1 = b, \quad a_{n+2} = da_{n+1} - a_n \quad \text{for } n = 0, 1, 2, \dots,$$

where  $a, b$  are non-zero integers,  $d$  is a real number. Find all  $d$  such that  $a_n$  is an integer for  $n = 0, 1, 2, \dots$

**271. 1 (Đoàn Thế Phiệt)** Find necessary and sufficient conditions with respect to  $m$  such that the system of equations

$$\begin{aligned} x^2 + y^2 + z^2 + xy - yz - zx &= 1, \\ y^2 + z^2 + yz &= 2, \\ z^2 + x^2 + zx &= m \end{aligned}$$

has a solution.

**272. 1 (Nguyễn Xuân Hùng)** Given are three externally tangent circles  $(O_1)$ ,  $(O_2)$ , and  $(O_3)$ . Let  $A, B, C$  be respectively the points of tangency of  $(O_1)$  and  $(O_3)$ ,  $(O_2)$  and  $(O_3)$ ,  $(O_1)$  and  $(O_2)$ . The common tangent of  $(O_1)$  and  $(O_2)$  meets  $C$  and  $(O_3)$  at  $M$  and  $N$ . Let  $D$  be the midpoint of  $MN$ . Prove that  $C$  is the center of one of the excircles of triangle  $ABD$ .

**272. 2 (Trịnh Bằng Giang)** Let  $ABCD$  be a convex quadrilateral such that  $AB + CD = BC + DA$ . Find the locus of points  $M$  interior to quadrilateral  $ABCD$  such that the sum of the distances from  $M$  to  $AB$  and  $CD$  is equal to the sum of the distances from  $M$  to  $BC$  and  $DA$ .

**272. 3 (Hồ Quang Vinh)** Let  $M$  and  $m$  be the greatest and smallest numbers in the set of positive numbers  $a_1, a_2, \dots, a_n$ ,  $n \geq 2$ . Prove that

$$\left(\sum_{i=1}^n a_i\right) \left(\sum_{i=1}^n \frac{1}{a_i}\right) \leq n^2 + \frac{n(n-1)}{2} \left(\sqrt{\frac{M}{m}} - \sqrt{\frac{m}{M}}\right)^2.$$

**272. 4 (Nguyễn Hữu Dự)** Find all primes  $p$  such that

$$f(p) = (2+3) - (2^2+3^2) + (2^3+3^3) - \dots - (2^{p-1}+3^{p-1}) + (2^p+3^p)$$

is divisible by 5.

**274. 1 (Đào Mạnh Thắng)** Let  $p$  be the semiperimeter and  $R$  the circumradius of triangle  $ABC$ . Furthermore, let  $D, E, F$  be the excenters. Prove that

$$DE^2 + EF^2 + FD^2 \geq 8\sqrt{3}pR,$$

and determine the equality case.

**274. 2 (Đoàn Thế Phiệt)** Determine the positive root of the equation

$$x \ln\left(1 + \frac{1}{x}\right)^{1+\frac{1}{x}} - x^3 \ln\left(1 + \frac{1}{x^2}\right)^{1+\frac{1}{x^2}} = 1 - x.$$

**274. 3 (N.Khánh Nguyễn)** Let  $ABCD$  be a cyclic quadrilateral. Points  $M, N, P,$  and  $Q$  are chosen on the sides  $AB, BC, CD,$  and  $DA,$  respectively, such that  $MA/MB = PD/PC = AD/BC$  and  $QA/QD = NB/NC = AB/CD.$  Prove that  $MP$  is perpendicular to  $NQ.$

**274. 4 (Nguyễn Hòa Liễu)** Prove the inequality for  $x \in \mathbb{R}:$

$$\frac{1 + 2x \arctan x}{2 + \ln(1 + x^2)^2} \geq \frac{1 + e^{\frac{x}{2}}}{3 + e^x}.$$

**275. 1 (Trần Hồng Sơn)** Let  $x, y, z$  be real numbers in the interval  $[-2, 2],$  prove the inequality

$$2(x^6 + y^6 + z^6) - (x^4y^2 + y^4z^2 + z^4x^2) \leq 192.$$

**276. 1 (Vũ Đức Cảnh)** Find the maximum value of the expression

$$f = \frac{a^3 + b^3 + c^3}{abc},$$

where  $a, b, c$  are real numbers lying in the interval  $[1, 2].$

**276. 2 (Hồ Quang Vinh)** Given a triangle  $ABC$  with sides  $BC = a, CA = b,$  and  $AB = c.$  Let  $R$  and  $r$  be the circumradius and inradius of the triangle, respectively. Prove that

$$\frac{a^3 + b^3 + c^3}{abc} \geq 4 - \frac{2r}{R}.$$

**276. 3 (Phạm Hoàng Hà)** Given a triangle  $ABC,$  let  $P$  be a point on the side  $BC,$  let  $H, K$  be the orthogonal projections of  $P$  onto  $AB, AC$  respectively. Points  $M, N$  are chosen on  $AB, AC$  such that  $PM \parallel AC$  and  $PN \parallel AB.$  Compare the areas of triangles  $PHK$  and  $PMN.$

**276. 4 (Đỗ Thanh Hân)** How many 6-digit natural numbers exist with the distinct digits and two arbitrary consecutive digits can not be simultaneously odd numbers?

**277. 1 (Nguyễn Hối)** The incircle with center  $O$  of a triangle touches the sides  $AB, AC,$  and  $BC$  respectively at  $D, E,$  and  $F.$  The escribed circle of triangle  $ABC$  in the angle  $A$  has center  $Q$  and touches the side  $BC$  and the rays  $AB, AC$  respectively at  $K, H,$  and  $I.$  The line  $DE$  meets the rays  $BO$

and  $CO$  respectively at  $M$  and  $N$ . The line  $HI$  meets the rays  $BQ$  and  $CQ$  at  $R$  and  $S$ , respectively. Prove that

$$\text{a) } \triangle FMN = \triangle KRS, \quad \text{b) } \frac{IS}{AB} = \frac{SR}{BC} = \frac{RH}{CA}.$$

**277. 2 (Nguyễn Đức Huy)** Find all rational numbers  $p, q, r$  such that

$$p \cos \frac{\pi}{7} + q \cos \frac{2\pi}{7} + r \cos \frac{3\pi}{7} = 1.$$

**277. 3 (Nguyễn Xuân Hùng)** Let  $ABCD$  be a bicentric quadrilateral inscribed in a circle with center  $I$  and circumscribed about a circle with center  $O$ . A line through  $I$ , parallel to a side of  $ABCD$ , intersects its two opposite sides at  $M$  and  $N$ . Prove that the length of  $MN$  does not depend on the choice of side to which the line is parallel.

**277. 4 (Đinh Thành Trung)** Let  $x \in (0, \pi)$  be real number and suppose that  $\frac{x}{\pi}$  is not rational. Define

$$S_1 = \sin x, \quad S_2 = \sin x + \sin 2x, \quad \dots, \quad S_n = \sin x + \sin 2x + \dots + \sin nx.$$

Let  $t_n$  be the number of negative terms in the sequence  $S_1, S_2, \dots, S_n$ . Prove that  $\lim_{n \rightarrow \infty} \frac{t_n}{n} = \frac{x}{2\pi}$ .

**279. 1 (Nguyễn Hữu Bằng)** Find all natural numbers  $a > 1$ , such that if  $p$  is a prime divisor of  $a$  then the number of all divisors of  $a$  which are relatively prime to  $p$ , is equal to the number of the divisors of  $a$  that are not relatively prime to  $p$ .

**279. 2 (Lê Duy Ninh)** Prove that for all real numbers  $a, b, x, y$  satisfying  $x + y = a + b$  and  $x^4 + y^4 = a^4 + b^4$  then  $x^n + y^n = a^n + b^n$  for all  $n \in \mathbb{N}$ .

**279. 3 (Nguyễn Hữu Phước)** Given an equilateral triangle  $ABC$ , find the locus of points  $M$  interior to  $ABC$  such that if the orthogonal projections of  $M$  onto  $BC, CA$  and  $AB$  are  $D, E$ , and  $F$ , respectively, then  $AD, BE$ , and  $CF$  are concurrent.

**279. 4 (Nguyễn Minh Hà)** Let  $M$  be a point in the interior of triangle  $ABC$  and let  $X, Y, Z$  be the reflections of  $M$  across the sides  $BC, CA$ , and  $AB$ , respectively. Prove that triangles  $ABC$  and  $XYZ$  have the same centroid.

**279. 5 (Vũ Đức Sơn)** Find all positive integers  $n$  such that  $n < t_n$ , where  $t_n$  is the number of positive divisors of  $n^2$ .

**279. 6 (Trần Nam Dũng)** Find the maximum value of the expression

$$\frac{x}{1+x^2} + \frac{y}{1+y^2} + \frac{z}{1+z^2},$$

where  $x, y, z$  are real numbers satisfying the condition  $x + y + z = 1$ .

**279. 7 (Hoàng Hoa Trại)** Given are three concentric circles with center  $O$ , and radii  $r_1 = 1$ ,  $r_2 = \sqrt{2}$ , and  $r_3 = \sqrt{5}$ . Let  $A, B, C$  be three non-collinear points lying respectively on these circles and let  $F$  be the area of triangle  $ABC$ . Prove that  $F \leq 3$ , and determine the side lengths of triangle  $ABC$ .

**281. 1 (Nguyễn Xuân Hùng)** Let  $P$  be a point exterior to a circle with center  $O$ . From  $P$  construct two tangents touching the circle at  $A$  and  $B$ . Let  $Q$  be a point, distinct from  $P$ , on the circle. The tangent at  $Q$  of the circle intersects  $AB$  and  $AC$  at  $E$  and  $F$ , respectively. Let  $BC$  intersect  $OE$  and  $OF$  at  $X$  and  $Y$ , respectively. Prove that  $XY/EF$  is a constant when  $P$  varies on the circle.

**281. 2 (Hồ Quang Vinh)** In a triangle  $ABC$ , let  $BC = a$ ,  $CA = b$ ,  $AB = c$  be the sides,  $r$ ,  $r_a$ ,  $r_b$ , and  $r_c$  be the inradius and exradii. Prove that

$$\frac{abc}{r} \geq \frac{a^3}{r_a} + \frac{b^3}{r_b} + \frac{c^3}{r_c}.$$

**283. 1 (Trần Hồng Sơn)** Simplify the expression

$$\sqrt{x(4-y)(4-z)} + \sqrt{y(4-z)(4-x)} + \sqrt{z(4-x)(4-y)} - \sqrt{xyz},$$

where  $x, y, z$  are positive numbers such that  $x + y + z + \sqrt{xyz} = 4$ .

**283. 2 (Nguyễn Phước)** Let  $ABCD$  be a convex quadrilateral,  $M$  be the midpoint of  $AB$ . Point  $P$  is chosen on the segment  $AC$  such that lines  $MP$  and  $BC$  intersect at  $T$ . Suppose that  $Q$  is on the segment  $BD$  such that  $BQ/QD = AP/PC$ . Prove that the line  $TQ$  has a fixed point when  $P$  moves on the segment  $AC$ .

**284. 1 (Nguyễn Hữu Bằng)** Given an integer  $n > 0$  and a prime  $p > n+1$ , prove or disprove that the following equation has integer solutions:

$$1 + \frac{x}{n+1} + \frac{x^2}{2n+1} + \cdots + \frac{x^p}{pn+1} = 0.$$

**284. 2 (Lê Quang Năm)** Let  $x, y$  be real numbers such that

$$(x + \sqrt{1+y^2})(y + \sqrt{1+x^2}) = 1,$$

prove that

$$(x + \sqrt{1+x^2})(y + \sqrt{1+y^2}) = 1.$$

**284. 3 (Nguyễn Xuân Hùng)** The internal angle bisectors  $AD$ ,  $BE$ , and  $CF$  of a triangle  $ABC$  meet at point  $Q$ . Prove that if the inradii of triangles  $AQF$ ,  $BQD$ , and  $CQE$  are equal then triangle  $ABC$  is equilateral.

**284. 4 (Trần Nam Dũng)** Disprove that there exists a polynomial  $p(x)$  of degree greater than 1 such that if  $p(x)$  is an integer then  $p(x + 1)$  is also an integer for  $x \in \mathbb{R}$ .

**285. 1 (Nguyễn Duy Liên)** Given an odd natural number  $p$  and integers  $a, b, c, d, e$  such that  $a + b + c + d + e$  and  $a^2 + b^2 + c^2 + d^2 + e^2$  are all divisible by  $p$ . Prove that  $a^5 + b^5 + c^5 + d^5 + e^5 - 5abcde$  is also divisible by  $p$ .

**285. 2 (Vũ Đức Cảnh)** Prove that if  $x, y \in \mathbb{R}^*$  then

$$\frac{2x^2 + 3y^2}{2x^3 + 3y^3} + \frac{2y^2 + 3x^2}{2y^3 + 3x^3} \leq \frac{4}{x + y}.$$

**285. 3 (Nguyễn Hữu Phước)** Let  $P$  be a point in the interior of triangle  $ABC$ . Rays  $AP$ ,  $BP$ , and  $CP$  intersect the sides  $BC$ ,  $CA$ , and  $AB$  at  $D$ ,  $E$ , and  $F$ , respectively. Let  $K$  be the point of intersection of  $DE$  and  $CM$ ,  $H$  be the point of intersection of  $DF$  and  $BM$ . Prove that  $AD$ ,  $BK$  and  $CH$  are concurrent.

**285. 4 (Trần Tuấn Anh)** Let  $a, b, c$  be non-negative real numbers, determine all real numbers  $x$  such that the following inequality holds:

$$\begin{aligned} [a^2 + b^2 + (x - 1)c^2][a^2 + c^2 + (x - 1)b^2][b^2 + c^2 + (x - 1)a^2] \\ \leq (a^2 + xbc)(b^2 + xac)(c^2 + xab). \end{aligned}$$

**285. 5 (Trương Cao Dũng)** Let  $O$  and  $I$  be the circumcenter and incenter of a triangle  $ABC$ . Rays  $AI$ ,  $BI$ , and  $CI$  meet the circumcircle at  $D$ ,  $E$ , and  $F$ , respectively. Let  $R_a$ ,  $R_b$ , and  $R_c$  be the radii of the escribed circles of  $\triangle ABC$ , and let  $R_d$ ,  $R_e$ , and  $R_f$  be the radii of the escribed circles of triangle  $DEF$ . Prove that

$$R_a + R_b + R_c \leq R_d + R_e + R_f.$$

**285. 6 (Đỗ Quang Dương)** Determine all integers  $k$  such that the sequence defined by  $a_1 = 1$ ,  $a_{n+1} = 5a_n + \sqrt{ka_n^2 - 8}$  for  $n = 1, 2, 3, \dots$  includes only integers.

**286. 1 (Trần Hồng Sơn)** Solve the equation

$$18x^2 - 18x\sqrt{x} - 17x - 8\sqrt{x} - 2 = 0.$$

**286. 2 (Phạm Hùng)** Let  $ABCD$  be a square. Points  $E, F$  are chosen on  $CB$  and  $CD$ , respectively, such that  $BE/BC = k$ , and  $DF/DC = (1 - k)/(1 + k)$ , where  $k$  is a given number,  $0 < k < 1$ . Segment  $BD$  meets  $AE$  and  $AF$  at  $H$  and  $G$ , respectively. The line through  $A$ , perpendicular to  $EF$ , intersects  $BD$  at  $P$ . Prove that  $PG/PH = DG/BH$ .

**286. 3 (Vũ Đình Hòa)** In a convex hexagon, the segment joining two of its vertices, dividing the hexagon into two quadrilaterals is called a *principal diagonal*. Prove that in every convex hexagon, in which the length of each side is equal to 1, there exists a principal diagonal with length not greater than 2 and there exists a principal diagonal with length greater than  $\sqrt{3}$ .

**286. 4 (Đỗ Bá Chủ)** Prove that in any acute or right triangle  $ABC$  the following inequality holds:

$$\tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2} + \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2} \geq \frac{10\sqrt{3}}{9}.$$

**286. 5 (Trần Tuấn Điệp)** In triangle  $ABC$ , no angle exceeding  $\frac{\pi}{2}$ , and each angle is greater than  $\frac{\pi}{4}$ . Prove that

$$\cot A + \cot B + \cot C + 3 \cot A \cot B \cot C \leq 4(2 - \sqrt{2}).$$

**287. 1 (Trần Nam Dũng)** Suppose that  $a, b$  are positive integers such that  $2a - 1, 2b - 1$  and  $a + b$  are all primes. Prove that  $a^b + b^a$  and  $a^a + b^b$  are not divisible by  $a + b$ .

**287. 2 (Phạm Đình Trường)** Let  $ABCD$  be a square in which the two diagonals intersect at  $E$ . A line through  $A$  meets  $BC$  at  $M$  and intersects  $CD$  at  $N$ . Let  $K$  be the intersection point of  $EM$  and  $BN$ . Prove that  $CK \perp BN$ .

**287. 3 (Nguyễn Xuân Hùng)** Let  $ABC$  be a right isosceles triangle,  $\angle A = 90^\circ$ ,  $I$  be the incenter of the triangle,  $M$  be the midpoint of  $BC$ . Let  $MI$  intersect  $AB$  at  $N$  and  $E$  be the midpoint of  $IN$ . Furthermore,  $F$  is chosen on side  $BC$  such that  $FC = 3FB$ . Suppose that the line  $EF$  intersects  $AB$  and  $AC$  at  $D$  and  $K$ , respectively. Prove that  $\triangle ADK$  is isosceles.

**287. 4 (Hoàng Hoa Trại)** Given a positive integer  $n$ , and  $w$  is the sum of  $n$  first integers. Prove that the equation

$$x^3 + y^3 + z^3 + t^3 = 2w^3 - 1$$

has infinitely many integer solutions.

**288. 1 (Vũ Đức Cảnh)** Find necessary and sufficient conditions for  $a, b, c$  for which the following equation has no solutions:

$$a(ax^2 + bx + c)^2 + b(ax^2 + bx + c) + c = x.$$

**288. 2 (Phạm Ngọc Quang)** Let  $ABCD$  be a cyclic quadrilateral,  $P$  be a variable point on the arc  $BC$  not containing  $A$ , and  $F$  be the foot of the perpendicular from  $C$  onto  $AB$ . Suppose that  $\triangle MEF$  is equilateral, calculate  $IK/R$ , where  $I$  is the incenter of triangle  $ABC$  and  $K$  the intersection (distinct from  $A$ ) of ray  $AI$  and the circumcircle of radius  $R$  of triangle  $ABC$ .

**288. 3 (Nguyễn Văn Thông)** Given a prime  $p > 2$  such that  $p - 2$  is divisible by 3. Prove that the set of integers defined by  $y^2 - x^3 - 1$ , where  $x, y$  are non-negative integers smaller than  $p$ , has at most  $p - 1$  elements divisible by  $p$ .

**289. 1 (Thái Nhật Phương)** Let  $ABC$  be a right isosceles triangle with  $A = 90^\circ$ . Let  $M$  be the midpoint of  $BC$ ,  $G$  be a point on side  $AB$  such that  $GB = 2GA$ . Let  $GM$  intersect  $CA$  at  $D$ . The line through  $M$ , perpendicular to  $CG$  at  $E$ , intersects  $AC$  at  $K$ . Finally, let  $P$  be the point of intersection of  $DE$  and  $GK$ . Prove that  $DE = BC$  and  $PG = PE$ .

**289. 2 (Hồ Quang Vinh)** Given a convex quadrilateral  $ABCD$ , let  $M$  and  $N$  be the midpoints of  $AD$  and  $BC$ , respectively,  $P$  be the point of intersection of  $AN$  and  $BM$ , and  $Q$  the intersection point of  $DN$  and  $CM$ . Prove that

$$\frac{PA}{PN} + \frac{PB}{PM} + \frac{QC}{QM} + \frac{QD}{QN} \geq 4,$$

and determine when equality holds.

**290. 1 (Nguyễn Song Minh)** Given  $x, y, z, t \in \mathbb{R}$  and real polynomial

$$F(x, y, z, t) = 9(x^2y^2 + y^2z^2 + z^2t^2 + t^2x^2) + 6xz(y^2 + t^2) - 4xyzt.$$

- Prove that the polynomial can be factored into the product of two quadratic polynomials.
- Find the minimum value of the polynomial  $F$  if  $xy + zt = 1$ .

**290. 2 (Phạm Hoàng Hà)** Let  $M$  be a point on the internal angle bisector  $AD$  of triangle  $ABC$ ,  $M$  distinct from  $A, D$ . Ray  $AM$  intersects side  $AC$  at  $E$ , ray  $CM$  meets side  $AB$  at  $F$ . Prove that if

$$\frac{1}{AB^2} + \frac{1}{AE^2} = \frac{1}{AC^2} + \frac{1}{AF^2}$$

then  $\triangle ABC$  is isosceles.

**290.3 (Đỗ Anh)** Consider a triangle  $ABC$  and its incircle. The internal angle bisector  $AD$  and median  $AM$  intersect the incircle again at  $P$  and  $Q$ , respectively. Compare the lengths of  $DP$  and  $MQ$ .

**290.4 (Nguyễn Duy Liên)** Find all pairs of integers  $(a, b)$  such that  $a + b^2$  divides  $a^2b - 1$ .

**290.5 (Đinh Thành Trung)** Determine all real functions  $f(x), g(x)$  such that  $f(x) - f(y) = \cos(x + y) \cdot g(x - y)$  for all  $x, y \in \mathbb{R}$ .

**290.6 (Nguyễn Minh Đức)** Find all real numbers  $a$  such that the system of equations has real solutions in  $x, y, z$ :

$$\begin{aligned}\sqrt{x-1} + \sqrt{y-1} + \sqrt{z-1} &= a - 1, \\ \sqrt{x+1} + \sqrt{y+1} + \sqrt{z+1} &= a + 1.\end{aligned}$$

**290.7 (Đoàn Kim Sang)** Given a positive integer  $n$ , find the number of positive integers, not exceeding  $n(n+1)(n+2)$ , which are divisible by  $n$ ,  $n+1$ , and  $n+2$ .

**291.1 (Bùi Minh Duy)** Given three distinct numbers  $a, b, c$  such that

$$\frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b} = 0,$$

prove that any two of the numbers have different signs.

**291.2 (Đỗ Thanh Hân)** Given three real numbers  $x, y, z$  that satisfy the conditions  $0 < x < y \leq z \leq 1$  and  $3x + 2y + z \leq 4$ . Find the maximum value of the expression  $3x^3 + 2y^2 + z^2$ .

**291.3 (Vi Quốc Dũng)** Given a circle of center  $O$  and two points  $A, B$  on the circle. A variable circle through  $A, B$  has center  $Q$ . Let  $P$  be the reflection of  $Q$  across the line  $AB$ . Line  $AP$  intersects the circle  $O$  again at  $E$ , while line  $BE$ ,  $E$  distinct from  $B$ , intersects the circle  $Q$  again at  $F$ . Prove that  $F$  lies on a fixed line when circle  $Q$  varies.

**291.4 (Vũ Đức Sơn)** Find all functions  $f : \mathbb{Q} \rightarrow \mathbb{Q}$  such that

$$f(f(x) + y) = x + f(y) \quad \text{for } x, y \in \mathbb{Q}.$$

**291.5 (Nguyễn Văn Thông)** Find the maximum value of the expression

$$x^2(y-z) + y^2(z-y) + z^2(1-z),$$

where  $x, y, z$  are real numbers such that  $0 \leq x \leq y \leq z \leq 1$ .

**291. 6 (Vũ Thành Long)** Given an acute-angled triangle  $ABC$  with side lengths  $a, b, c$ . Let  $R, r$  denote its circumradius and inradius, respectively, and  $F$  its area. Prove the inequality

$$ab + bc + ca \geq 2R^2 + 2Rr + \frac{8}{\sqrt{3}}F.$$

**292. 1 (Thái Nhật Phương, Trần Hà)** Let  $x, y, z$  be positive numbers such that  $xyz = 1$ , prove the inequality

$$\frac{x^2}{x + y + y^3z} + \frac{y^2}{y + z + z^3x} + \frac{z^2}{z + x + x^3y} \leq 1.$$

**292. 2 (Phạm Ngọc Bội)** Let  $p$  be an odd prime, let  $a_1, a_2, \dots, a_{p-1}$  be  $p - 1$  integers that are not divisible by  $p$ . Prove that among the sums  $T = k_1a_1 + k_2a_2 + \dots + k_{p-1}a_{p-1}$ , where  $k_i \in \{-1, 1\}$  for  $i = 1, 2, \dots, p - 1$ , there exists at least a sum  $T$  divisible by  $p$ .

**292. 3 (Ha Vu Anh)** Given are two circles  $\Gamma_1$  and  $\Gamma_2$  intersecting at two distinct points  $A, B$  and a variable point  $P$  on  $\Gamma_1$ ,  $P$  distinct from  $A$  and  $B$ . The lines  $PA, PB$  intersect  $\Gamma_2$  at  $D$  and  $E$ , respectively. Let  $M$  be the midpoint of  $DE$ . Prove that the line  $MP$  has a fixed point.

**294. 1 (Phùng Trọng Thực)** Triangle  $ABC$  is inscribed in a circle of center  $O$ . Let  $M$  be a point on side  $AC$ ,  $M$  distinct from  $A, C$ , the line  $BM$  meets the circle again at  $N$ . Let  $Q$  be the intersection of a line through  $A$  perpendicular to  $AB$  and a line through  $N$  perpendicular to  $NC$ . Prove that the line  $QM$  has a fixed point when  $M$  varies on  $AC$ .

**294. 2 (Trần Xuân Bang)** Let  $A, B$  be the intersections of circle  $O$  of radius  $R$  and circle  $O'$  of radius  $R'$ . A line touches circle  $O$  and  $O'$  at  $T$  and  $T'$ , respectively. Prove that  $B$  is the centroid of triangle  $ATT'$  if and only if

$$OO' = \frac{\sqrt{3}}{2}(R + R').$$

**294. 3 (Vũ Trí Đức)** If  $a, b, c$  are positive real numbers such that  $ab + bc + ca = 1$ , find the minimum value of the expression  $w(a^2 + b^2) + c^2$ , where  $w$  is a positive real number.

**294. 4 (Lê Quang Năm)** Let  $p$  be a prime greater than 3, prove that  $\binom{p-1}{2001p^2-1} - 1$  is divisible by  $p^4$ .

**294. 5 (Trương Ngọc Đắc)** Let  $x, y, z$  be positive real numbers such that  $x = \max\{x, y, z\}$ , find the minimum value of

$$\frac{x}{y} + \sqrt{1 + \frac{y}{z}} + \sqrt[3]{1 + \frac{z}{x}}.$$

**294. 6 (Phạm Hoàng Hà)** The sequence  $(a_n)$ ,  $n = 1, 2, 3, \dots$ , is defined by  $a_n = \frac{1}{n^2(n+2)\sqrt{n+1}}$  for  $n = 1, 2, 3, \dots$ . Prove that

$$a_1 + a_2 + \dots + a_n < \frac{1}{2\sqrt{2}} \quad \text{for } n = 1, 2, 3, \dots$$

**294. 7 (Vũ Huy Hoàng)** Given are a circle  $O$  of radius  $R$ , and an odd natural number  $n$ . Find the positions of  $n$  points  $A_1, A_2, \dots, A_n$  on the circle such that the sum  $A_1A_2 + A_2A_3 + \dots + A_{n-1}A_n + A_nA_1$  is a minimum.

**295. 1 (Trần Tuyết Thanh)** Solve the equation

$$x^2 - x - 1000\sqrt{1 + 8000x} = 1000.$$

**295. 2 (Phạm Đình Trường)** Let  $A_1A_2A_3A_4A_5A_6$  be a convex hexagon with parallel opposite sides. Let  $B_1, B_2$ , and  $B_3$  be the points of intersection of pairs of diagonals  $A_1A_4$  and  $A_2A_5$ ,  $A_2A_5$  and  $A_3A_6$ ,  $A_3A_6$  and  $A_1A_4$ , respectively. Let  $C_1, C_2, C_3$  be respectively the midpoints of the segments  $A_3A_6$ ,  $A_1A_4$ ,  $A_2A_5$ . Prove that  $B_1C_1, B_2C_2, B_3C_3$  are concurrent.

**295. 3 (Bùi Thế Hùng)** Let  $A, B$  be respectively the greatest and smallest numbers from the set of  $n$  positive numbers  $x_1, x_2, \dots, x_n$ ,  $n \geq 2$ . Prove that

$$A < \frac{(x_1 + x_2 + \dots + x_n)^2}{x_1 + 2x_2 + \dots + nx_n} < 2B.$$

**295. 4 (Trần Tuấn Anh)** Prove that if  $x, y, z > 0$  then

- $(x + y + z)^3(y + z - x)(z + x - y)(x + y - z) \leq 27x^3y^3z^3$ ,
- $(x^2 + y^2 + z^2)(y + z - x)(z + x - y)(x + y - z) \leq xyz(yz + zx + xy)$ ,
- $(x + y + z)[2(yz + zx + xy) - (x^2 + y^2 + z^2)] \leq 9xyz$ .

**295. 5 (Vũ Thị Huệ Phương)** Find all functions  $f : \mathbb{D} \rightarrow \mathbb{D}$ , where  $\mathbb{D} = [1, +\infty)$  such that

$$f(xf(y)) = yf(x) \quad \text{for } x, y \in \mathbb{D}.$$

**295. 6 (Nguyễn Viết Long)** Given an even natural number  $n$ , find all polynomials  $p_n(x)$  of degree  $n$  such that

- all the coefficients of  $p_n(x)$  are elements from the set  $\{0, -1, 1\}$  and  $p_n(0) \neq 0$ ;
- there exists a polynomial  $q(x)$  with coefficients from the set  $\{0, -1, 1\}$  such that  $p_n(x) \equiv (x^2 - 1)q(x)$ .

**296. 1 (Thới Ngọc Anh)** Prove that

$$\frac{1}{6} < \frac{3 - \underbrace{\sqrt{6 + \sqrt{6 + \cdots + \sqrt{6}}}}_{n \text{ times}}}{3 - \underbrace{\sqrt{6 + \sqrt{6 + \cdots + \sqrt{6}}}}_{(n-1) \text{ times}}} < \frac{5}{27},$$

where there are  $n$  radical signs in the expression of the numerator and  $n - 1$  ones in the expression of the denominator.

**296. 2 (Vi Quốc Dũng)** Let  $ABC$  be a triangle and  $M$  the midpoint of  $BC$ . The external angle bisector of  $A$  meets  $BC$  at  $D$ . The circumcircle of triangle  $ADM$  intersects line  $AB$  and line  $AC$  at  $E$  and  $F$ , respectively. If  $N$  is the midpoint of  $EF$ , prove that  $MN \parallel AD$ .

**296. 3 (Nguyễn Văn Hiến)** Let  $k, n \in \mathbb{N}$  such that  $k < n$ . Prove that

$$\frac{(n+1)^{n+1}}{(k+1)^{k+1}(n-k+1)^{n-k+1}} < \frac{n!}{k!(n-k)!} < \frac{n^n}{k^k(n-k+1)^{n-k}}.$$

**297. 1 (Nguyễn Hữu Phước)** Given a circle with center  $O$  and diameter  $EF$ . Points  $N, P$  are chosen on line  $EF$  such that  $ON = OP$ . From a point  $M$  interior to the circle, not lying on  $EF$ , draw  $MN$  intersecting the circle at  $A$  and  $C$ , draw  $MP$  meeting the circle at  $B$  and  $D$  such that  $B$  and  $O$  are on different sides of  $AC$ . Let  $K$  be the point of intersection of  $OB$  and  $AC$ ,  $Q$  the point of intersection of  $EF$  and  $CD$ . Prove that lines  $KQ, BD, AO$  are concurrent.

**297. 2 (Trần Nam Dũng)** Let  $a$  and  $b$  two relatively prime numbers. Prove that there exist exactly  $\frac{1}{2}(ab - a - b + 1)$  natural numbers that can not be written in the form  $ax + by$ , where  $x$  and  $y$  are non-negative integers.

**297. 3 (Lê Quốc Hán)** The circle with center  $I$  and radius  $r$  touches the sides  $BC = a$ ,  $CA = b$ , and  $AB = c$  of triangle  $ABC$  at  $M, N$ , and  $P$ , respectively. Let  $F$  be the area of triangle  $ABC$  and  $h_a, h_b, h_c$  be the lengths of the altitudes of  $\triangle ABC$ . Prove that

$$\begin{aligned} \text{a) } & 4F^2 = ab \cdot MN^2 + bc \cdot NP^2 + ca \cdot PM^2; \\ \text{b) } & \frac{MN^2}{h_a h_b} + \frac{NP^2}{h_b h_c} + \frac{PM^2}{h_c h_a} = 1. \end{aligned}$$

**298. 1 (Phạm Hoàng Hà)** Let  $P$  be the midpoint of side  $BC$  of triangle  $ABC$  and let  $BE, CF$  be two altitudes of the triangle. The line through  $A$ , perpendicular to  $PF$ , meets  $CF$  at  $M$ ; the line through  $A$ , perpendicular to  $PE$ , intersects  $BE$  at  $N$ . Let  $K$  and  $G$  be respectively the midpoints of  $BM$  and  $CN$ . Finally, let  $H$  be the intersection of  $KF$  and  $GE$ . Prove that  $AH$  is perpendicular to  $EF$ .

**298. 2 (Phạm Đình Trường)** Let  $ABCD$  be a square. Points  $E$  and  $F$  are chosen on sides  $AB$  and  $CD$ , respectively, such that  $AE = CF$ . Let  $AD$  intersect  $CE$  and  $BF$  at  $M$  and  $N$ , respectively. Suppose that  $P$  is the intersection of  $BM$  and  $CN$ , find the locus of  $P$  when  $E$  and  $F$  move on the side  $AB$  and  $CD$ , respectively.

**298. 3 (Nguyễn Minh Hà)** Let  $ABCD$  be a convex quadrilateral, let  $AB$  intersect  $CD$  at  $E$ ;  $AD$  intersects  $BC$  at  $F$ . Prove that the midpoints of line segments  $AB, CD$ , and  $EF$  are collinear.

**298. 4 (Nguyễn Minh Hà)** Given a cyclic quadrilateral  $ABCD$ ,  $M$  is any point in the plane. Let  $X, Y, Z, T, U, V$  be the orthogonal projections of  $M$  on the lines  $AB, CD, AC, DB, AD$ , and  $BC$ . Let  $E, F, G$  be the midpoints of  $XY, ZT$ , and  $UV$ . Prove that  $E, F$ , and  $G$  are collinear.

**300. 1 (Vũ Trí Đức)** Find the maximum and minimum values of the expression  $x\sqrt{1+y} + y\sqrt{1+x}$ , where  $x, y$  are non-negative real numbers such that  $x + y = 1$ .

**300. 2 (Nguyễn Xuân Hùng)** Let  $P$  be a point in the interior of triangle  $ABC$ . The incircle of triangle  $ABC$  is tangent to sides  $BC, CA$  and  $AB$  at  $D, E$ , and  $F$ , respectively. The incircle of triangle  $PBC$  touches the sides  $BC, CP$ , and  $PB$  at  $K, M$ , and  $N$ , respectively. Suppose that  $Q$  is the point of intersection of lines  $EM$  and  $FN$ . Prove that  $A, P, Q$  are collinear if and only if  $K$  coincides with  $D$ .

**300. 3 (Huỳnh Tấn Châu)** Determine all pairs of integers  $(m, n)$  such that

$$\frac{n}{m} = \frac{(m^2 - n^2)^{n/m} - 1}{(m^2 - n^2)^{n/m} + 1}.$$

**300. 4 (Võ Giang Giai, Mạnh Tú)** Prove that if  $a, b, c, d, e \geq 0$  then

$$\frac{a + b + c + d + e}{5} \geq \sqrt[5]{abcde} + \frac{q}{20},$$

where  $q = (\sqrt{a} - \sqrt{b})^2 + (\sqrt{b} - \sqrt{c})^2 + (\sqrt{c} - \sqrt{d})^2 + (\sqrt{d} - \sqrt{e})^2$ .

**301.1 (Lê Quang Nãm)** Find all pairs of integers  $(x, y)$  such that  $x^2 + xy + y^2 + 14x + 14y + 2018$  is divisible by 101.

**301.2 (Nguyễn Thế Bình)** Find smallest value of the expression

$$\frac{2}{ab} + \frac{1}{a^2 + b^2} + \frac{a^4 + b^4}{2}$$

where  $a, b$  are real positive numbers such that  $a + b = 1$ .

**301.3 (Đỗ Anh)** Suppose that  $a, b, c$  are side lengths of a triangle and  $0 \leq t \leq 1$ . Prove that

$$\sqrt{\frac{a}{b+c-ta}} + \sqrt{\frac{b}{c+a-tb}} + \sqrt{\frac{c}{a+b-tc}} \geq 2\sqrt{1+t}.$$

**301.4 (Nguyễn Trọng Tuấn)** The sequence  $(a_n)$  is defined by  $a_1 = 5, a_2 = 11$  and  $a_{n+1} = 2a_n - 3a_{n-1}$  for  $n = 2, 3, \dots$ . Prove that the sequence has indefinitely many positive and negative terms, and show that  $a_{2002}$  is divisible by 11.

**301.5 (Trần Xuân Đáng)** Find the maximum value of  $3(a+b+c) - 22abc$ , where  $a, b, c \in \mathbb{R}$  such that  $a^2 + b^2 + c^2 = 1$ .

**301.6 (Nguyễn Văn Tình)** Given is an equilateral triangle  $ABC$  with centroid  $G$ . A variable line through the centroid and intersects the side  $BC$ ,  $CA$ , and  $AB$  at  $M, N$ , and  $P$  respectively. Prove that  $GM^{-4} + GN^{-4} + GP^{-4}$  is a constant.

**301.7 (Lê Hào)** A convex quadrilateral  $ABCD$  is inscribed in a circle with center  $O$ , radius  $R$ . Let  $CD$  intersect  $AB$  at  $E$ , a line through  $E$  meets the lines  $AD$  and  $BC$  at  $P, Q$ . Prove that

$$\frac{1}{EP} + \frac{1}{EQ} \leq \frac{2EO}{EO^2 - R^2},$$

and determine when equality holds.

**306.1 (Phan Thị Mùi)** Prove that if  $x, y, z > 0$  and  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$  then

$$(x+y-z-1)(y+z-x-1)(z+x-y-1) \leq 8.$$

**306.2 (Trần Tuấn Anh)** Given an integer  $m \geq 4$ , find the maximum and minimum values of the expression  $ab^{m-1} + a^{m-1}b$ , where  $a, b$  are real numbers such that  $a + b = 1$  and  $0 \leq a, b \leq \frac{m-2}{m}$ .

**308. 1 (Lê Thị Anh Thư)** Find all integer solutions of the equation

$$4(a-x)(x-b) + b - a = y^2,$$

where  $a, b$  are given integers,  $a > b$ .

**308. 2 (Phan Thế Hải)** Given a convex quadrilateral  $ABCD$ ,  $E$  is the point of intersection of  $AB$  and  $CD$ , and  $F$  is the intersection of  $AD$  and  $BC$ . The diagonals  $AC$  and  $BD$  meet at  $O$ . Suppose that  $M, N, P, Q$  are the midpoints of  $AB, BC, CD$ , and  $DA$ . Let  $H$  be the intersection of  $OF$  and  $MP$ , and  $K$  the intersection of  $OE$  and  $NQ$ . Prove that  $HK \parallel EF$ .

**309. 1 (Vũ Hoàng Hiệp)** Given a positive integer  $n$ , find the smallest possible  $t = t(n)$  such that for all real numbers  $x_1, x_2, \dots, x_n$  we have

$$\sum_{k=1}^n (x_1 + x_2 + \dots + x_k)^2 \leq t(x_1^2 + x_2^2 + \dots + x_n^2).$$

**309. 2 (Lê Xuân Sơn)** Given a triangle  $ABC$ , prove that

$$\sin A \cos B + \sin B \cos C + \sin C \cos A \leq \frac{3\sqrt{3}}{4}.$$

**311. 1 (Nguyễn Xuân Hùng)** The chord  $PQ$  of the circumcircle of a triangle  $ABC$  meets its incircle at  $M$  and  $N$ . Prove that  $PQ \geq 2MN$ .

**311. 2 (Đàm Văn Nhi)** Given a convex quadrilateral  $ABCD$  with perpendicular diagonals  $AC$  and  $BD$ , let  $BC$  intersect  $AD$  at  $I$  and let  $AB$  meet  $CD$  at  $J$ . Prove that  $BDIJ$  is cyclic if and only if  $AB \cdot CD = AD \cdot BC$ .

**318. 1 (Đậu Thị Hoàng Oanh)** Prove that if  $2n$  is a sum of two distinct perfect square numbers (greater than 1) then  $n^2 + 2n$  is the sum of four perfect square numbers (greater than 1).

**318. 2 (Nguyễn Đễ)** Solve the system of equations

$$x^2(y+z)^2 = (3x^2 + x + 1)y^2z^2,$$

$$y^2(z+x)^2 = (4y^2 + y + 1)z^2x^2,$$

$$z^2(x+y)^2 = (5z^2 + z + 1)x^2y^2.$$

**318. 3 (Trần Việt Hùng)** A quadrilateral  $ABCD$  is inscribed in a circle such that the circle of diameter  $CD$  intersects the line segments  $AC, AD, BC, BD$  respectively at  $A_1, A_2, B_1, B_2$ , and the circle of diameter  $AB$  meets the line segments  $CA, CB, DA, DB$  respectively at  $C_1, C_2, D_1, D_2$ . Prove that there exists a circle that is tangent to the four lines  $A_1A_2, B_1B_2, C_1C_2$  and  $D_1D_2$ .

**319.1 (Dương Châu Dinh)** Prove the inequality

$$x^2y + y^2z + z^2x \leq x^3 + y^3 + z^3 \leq 1 + \frac{1}{2}(x^4 + y^4 + z^4),$$

where  $x, y, z$  are real non-negative numbers such that  $x + y + z = 2$ .

**319.2 (Tô Minh Hoàng)** Find all functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that

$$2(f(m^2 + n^2))^3 = f^2(m)f(n) + f^2(n)f(m)$$

for distinct  $m$  and  $n$ .

**319.3 (Trần Việt Anh)** Suppose that  $AD, BE$  and  $CF$  are the altitudes of an acute triangle  $ABC$ . Let  $M, N$ , and  $P$  be the intersection points of  $AD$  and  $EF$ ,  $BE$  and  $FD$ ,  $CF$  and  $DE$  respectively. Denote the area of triangle  $XYZ$  by  $F[XYZ]$ . Prove that

$$\frac{1}{F[ABC]} \leq \frac{F[MNP]}{F^2[DEF]} \leq \frac{1}{8 \cos A \cos B \cos C \cdot F[ABC]}.$$

**320.1 (Nguyễn Quang Long)** Find the maximum value of the function  $f = \sqrt{4x - x^3} + \sqrt{x + x^3}$  for  $0 \leq x \leq 2$ .

**320.2 (Vũ Đình Hòa)** Two circles of centers  $O$  and  $O'$  intersect at  $P$  and  $Q$  (see Figure). The common tangent, adjacent to  $P$ , of the two circles touches  $O$  at  $A$  and  $O'$  at  $B$ . The tangent of circle  $O$  at  $P$  intersects  $O'$  at  $C$ ; and the tangent of  $O'$  at  $P$  meets the circle  $O$  at  $D$ . Let  $M$  be the reflection of  $P$  across the midpoint of  $AB$ . The line  $AP$  intersects  $BC$  at  $E$  and the line  $BP$  meets  $AD$  at  $F$ . Prove that the hexagon  $AMBEQF$  is cyclic.

**320.3 (Hồ Quang Vinh)** Let  $R$  and  $r$  be the circumradius and inradius of triangle  $ABC$ ; the incircle touches the sides of the triangle at three points which form a triangle of perimeter  $p$ . Suppose that  $q$  is the perimeter of triangle  $ABC$ . Prove that  $r/R \leq p/q \leq \frac{1}{2}$ .

**321.1 (Lê Thanh Hải)** Prove that for all positive numbers  $a, b, c, d$

$$\begin{aligned} \text{a) } & \frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq \frac{a+b+c}{\sqrt[3]{abc}}; \\ \text{b) } & \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{d^2} + \frac{d^2}{a^2} \geq \frac{a+b+c+d}{\sqrt[4]{abcd}}. \end{aligned}$$

**321. 2 (Phạm Hoàng Hà)** Find necessary and sufficient conditions for which the system of equations

$$\begin{aligned}x^2 &= (2 + m)y^3 - 3y^2 + my, \\y^2 &= (2 + m)z^3 - 3z^2 + mz, \\z^2 &= (2 + m)x^3 - 3x^2 + mx\end{aligned}$$

has a unique solution.

**321. 3 (Trần Việt Anh)** Let  $m, n, p$  be three positive integers such that  $n + 1$  is divisible by  $m$ . Find a formula for the set of numbers  $(x_1, x_2, \dots, x_p)$  of  $p$  positive primes such that the sum  $x_1 + x_2 + \dots + x_p$  is divisible by  $m$ , with each number of the set not exceeding  $n$ .

**322. 1 (Nguyễn Như Hiền)** Given a triangle  $ABC$  with incenter  $I$ . The lines  $AI$  and  $DI$  intersect the circumcircle of triangle  $ABC$  again at  $H$  and  $K$ , respectively. Draw  $IJ$  perpendicular to  $BC$  at  $J$ . Prove that  $H, K$  and  $J$  are collinear.

**322. 2 (Trần Tuấn Anh)** Prove the inequality

$$\frac{1}{2} \left( \sum_{i=1}^n x_i + \sum_{i=1}^n \frac{1}{x_i} \right) \geq n - 1 + \frac{n}{\sum_{i=1}^n x_i},$$

where  $x_i$  ( $i = 1, 2, \dots, n$ ) are positive real numbers such that  $\sum_{i=1}^n x_i^2 = n$ , with  $n$  as an integer,  $n > 1$ .

**323. 1 (Nguyễn Đức Thuận)** Suppose that  $ABCD$  is a convex quadrilateral. Points  $E, F$  are chosen on the lines  $BC$  and  $AD$ , respectively, such that  $AE \parallel CD$  and  $CF \parallel AB$ . Prove that  $A, B, C, D$  are concyclic if and only if  $AECF$  has an incircle.

**323. 2 (Nguyễn Thế Phiệt)** Prove that for an acute triangle  $ABC$ ,

$$\cos A + \cos B + \cos C + \frac{1}{3}(\cos 3B + \cos 3C) \geq \frac{5}{6}.$$

**324. 1 (Trần Nam Dũng)** Find the greatest possible real number  $c$  such that we can always choose a real number  $x$  which satisfies the inequality  $\sin(mx) + \sin(nx) \geq c$  for each pair of positive integers  $m$  and  $n$ .

**325. 1 (Nguyễn Đăng Phát)** Given a convex hexagon inscribed in a circle such that the opposite sides are parallel. Prove that the sums of the lengths of each pair of opposite sides are equal if and only if the distances of the opposite sides are the same.

**325. 2 (Đinh Văn Khâm)** Given a natural number  $n$  and a prime  $p$ , how many sets of  $p$  natural numbers  $\{a_0, a_1, \dots, a_{p-1}\}$  are there such that

$$\text{a) } 1 \leq a_i \leq n \quad \text{for each } i = 0, 1, \dots, p-1,$$

$$\text{b) } [a_0, a_1, \dots, a_{p-1}] = p \min\{a_0, a_1, \dots, a_{p-1}\},$$

where  $[a_0, a_1, \dots, a_{p-1}]$  denotes the least common multiple of the numbers  $a_0, a_1, \dots, a_{p-1}$ ?

**327. 1 (Hoàng Trọng Hảo)** Let  $ABCD$  be a bicentric quadrilateral (i.e., it has a circumcircle of radius  $R$  and an incircle of radius  $r$ ). Prove that  $R \geq r\sqrt{2}$ .

**327. 2 (Vũ Đình Thế)** Two sequences  $(x_n)$  and  $(y_n)$  are defined by

$$\begin{aligned} x_{n+1} &= -2x_n^2 - 2x_n y_n + 8y_n^2, & x_1 &= -1, \\ y_{n+1} &= 2x_n^2 + 3x_n y_n - 2y_n^2, & y_1 &= 1 \end{aligned}$$

for  $n = 1, 2, 3, \dots$ . Find all primes  $p$  such that  $x_p + y_p$  is not divisible by  $p$ .

**328. 1 (Bùi Văn Chi)** Find all integer solutions  $(n, m)$  of the equation

$$(n+1)(2n+1) = 10m^2.$$

**328. 2 (Nguyễn Thị Minh)** Determine all positive integers  $n$  such that the polynomial of  $n+1$  terms

$$p(x) = x^{4n} + x^{4(n-1)} + \dots + x^8 + x^4 + 1$$

is divisible by the polynomial of  $n+1$  terms

$$q(x) = x^{2x} + x^{2(n-1)} + \dots + x^4 + x^2 + 1.$$

**328. 3 (Bùi Thế Hùng)** Find the smallest possible prime  $p$  such that  $[(3 + \sqrt{p})^{2n}] + 1$  is divisible by  $2^{n+1}$  for each natural number  $n$ , where  $[x]$  denotes the integral part of  $x$ .

**328. 4 (Hàn Ngọc Đức)** Find all real numbers  $a$  such that there exists a positive real number  $k$  and functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  which satisfy the inequality

$$\frac{f(x) + f(y)}{2} \geq f\left(\frac{x+y}{2}\right) + k|x-y|^a,$$

for all real numbers  $x, y$ .

**328. 5 (Vũ Hoàng Hiệp)** In space, let  $A_1, A_2, \dots, A_n$  be  $n$  distinct points. Prove that

$$\text{a) } \sum_{i=1}^n \angle A_i A_{i+1} A_{i+2} \geq \pi,$$

$$\text{b) } \sum_{i=1}^n \angle A_i Q A_{i+1} \leq (n-1)\pi,$$

where  $A_{n+i}$  is equal to  $A_i$  and  $Q$  is an arbitrary point distinct from  $A_1, A_2, \dots, A_n$ .

**329. 1 (Hoàng Ngọc Minh)** Find the maximum value of the expression

$$(a-b)^4 + (b-c)^4 + (c-a)^4,$$

for any real numbers  $1 \leq a, b, c \leq 2$ .

**331. 1 (Nguyễn Mạnh Tuấn)** Let  $x, y, z, w$  be rational numbers such that  $x + y + z + w = 0$ . Show that the number

$$\sqrt{(xy - zw)(yz - wx)(zx - yw)}$$

is also rational.

**331. 2 (Bùi Đình Thân)** Given positive reals  $a, b, c, x, y, z$  such that

$$a + b + c = 4 \quad \text{and} \quad ax + by + cz = xyz,$$

show that  $x + y + z > 4$ .

**331. 3 (Phạm Năng Khánh)** Given a triangle  $ABC$  and its angle bisector  $AM$ , the line perpendicular to  $BC$  at  $M$  intersects line  $AB$  at  $N$ . Prove that  $\angle BAC$  is a right angle if and only if  $MN = MC$ .

**331. 4 (Đào Tam)** Diagonals  $AC, BD$  of quadrilateral  $ABCD$  intersect at  $I$  such that  $IA = ID$  and  $\angle AID = 120^\circ$ . From point  $M$  on segment  $BC$ , draw  $MN \parallel AC$  and  $MQ \parallel BD$ ,  $N$  and  $Q$  are on  $AB$  and  $CD$ , respectively. Find the locus of circumcenter of triangle  $MNQ$  when  $M$  moves on line segment  $BC$ .

**331. 5 (Nguyễn Trọng Hiệp)** Let  $p, q$  be primes such that  $p > q > 2$ . Find all integers  $k$  such that the equation  $(px - qy)^2 = kxyz$  has integer solutions  $(x, y, z)$  with  $xy \neq 0$ .

**331.6 (Hàn Ngọc Đức)** Let a sequence  $(u_n)$ ,  $n = 1, 2, 3, \dots$ , be given defined by  $u_n = n^{2^n}$  for all  $n = 1, 2, \dots$ . Let

$$x_n = \frac{1}{u_1} + \frac{1}{u_2} + \dots + \frac{1}{u_n}.$$

Prove that the sequence  $(x_n)$  has a limit as  $n$  tends to infinity and that the limit is irrational.

**331.7 (Trần Tuấn Anh)** Find all positive integers  $n \geq 3$  such that the following inequality holds for all real numbers  $a_1, a_2, \dots, a_n$  (assume  $a_{n+1} = a_1$ )

$$\sum_{1 \leq i < j \leq n} (a_i - a_j)^2 \leq \left( \sum_{i=1}^n |a_i - a_{i+1}| \right)^2.$$

**332.1 (Nguyễn Văn Ái)** Find the remainder in the integer division of the number  $a^b + b^a$  by 5, where  $a = \overline{22\dots 2}$  with 2004 digits 2, and  $b = \overline{33\dots 3}$  with 2005 digits 3 (written in the decimal system).

**332.2 (Nguyễn Khánh Nguyên)** Suppose that  $ABC$  is an isosceles triangle with  $AB = AC$ . On the line perpendicular to  $AC$  at  $C$ , let point  $D$  such that points  $B, D$  are on different sides of  $AC$ . Let  $K$  be the intersection point of the line perpendicular to  $AB$  at  $B$  and the line passing through the midpoint  $M$  of  $CD$ , perpendicular to  $AD$ . Compare the lengths of  $KB$  and  $KD$ .

**332.3 (Phạm Văn Hoàng)** Consider the equation

$$x^2 - 2kxy^2 + k(y^3 - 1) = 0,$$

where  $k$  is some integer. Prove that the equation has integer solutions  $(x, y)$  such that  $x > 0, y > 0$  if and only if  $k$  is a perfect square.

**332.4 (Đỗ Văn Ta)** Solve the equation

$$\sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x - 5}}}} = 5.$$

**332.5 (Phạm Xuân Trinh)** Show that if  $a \geq 0$  then

$$\sqrt{a} + \sqrt[3]{a} + \sqrt[6]{a} \leq a + 2.$$

**332.6 (Bùi Văn Chi)** Let  $ABCD$  be a parallelogram with  $AB < BC$ . The bisector of angle  $\angle BAD$  intersects  $BC$  at  $E$ ; let  $O$  be the intersection point of the perpendicular bisectors of  $BD$  and  $CE$ . A line passing through  $C$  parallel to  $BD$  intersects the circle with center  $O$  and radius  $OC$  at  $F$ . Determine  $\angle AFC$ .

**332. 7 (Phan Hoàng Ninh)** Prove that the polynomial

$$p(x) = x^4 - 2003x^3 + (2004 + a)x^2 - 2005x + a$$

with  $a \in \mathbb{Z}$  has at most one integer solution. Furthermore, prove that it has no multiple integral root greater than 1.

**332. 8 (Phùng Văn Sử)** Prove that for any real numbers  $a, b, c$

$$(a^2 + 3)(b^2 + 3)(c^2 + 3) \geq \frac{4}{27}(3ab + 3bc + 3ca + abc)^2.$$

**332. 9 (Nguyễn Văn Thành)** Determine all functions  $f(x)$  defined on the interval  $(0, +\infty)$  which have a derivative at  $x = 1$  and that satisfy

$$f(xy) = \sqrt{x}f(y) + \sqrt{y}f(x)$$

for all positive real numbers  $x, y$ .

**332. 10 (Hoàng Ngọc Cảnh)** Let  $A_1A_2 \dots A_n$  be a  $n$ -gon inscribed in the unit circle; let  $M$  be a point on the minor arc  $A_1A_n$ . Prove that

$$\begin{aligned} \text{a) } & MA_1 + MA_3 + \dots + MA_{n-2} + MA_n < \frac{n}{\sqrt{2}} \quad \text{for } n \text{ odd;} \\ \text{b) } & MA_1 + MA_3 + \dots + MA_{n-3} + MA_{n-1} \leq \frac{n}{\sqrt{2}} \quad \text{for } n \text{ even.} \end{aligned}$$

When does equality hold?

**332. 11 (Đặng Thanh Hải)** Let  $ABC$  be an equilateral triangle with centroid  $O$ ;  $\ell$  is a line perpendicular to the plane  $(ABC)$  at  $O$ . For each point  $S$  on  $\ell$ , distinct from  $O$ , a pyramid  $SABC$  is defined. Let  $\phi$  be the dihedral angle between a lateral face and the base, let  $\gamma$  be the angle between two adjacent lateral faces of the pyramid. Prove that the quantity  $F(\phi, \gamma) = \tan^2 \phi [3 \tan^2(\gamma/2) - 1]$  is independent of the position of  $S$  on  $\ell$ .

**334. 1 (Đặng Như Tuấn)** Determine the sum

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{(n-1)n(n+1)} + \dots + \frac{1}{23 \cdot 24 \cdot 25}.$$

**334. 2 (Nguyễn Phước)** Let  $ABC$  be a triangle with angle  $A$  not being right,  $B \neq 135^\circ$ . Let  $M$  be the midpoint of  $BC$ . A right isosceles triangle  $ABD$  is outwardly erected on the side  $BC$  as base. Let  $E$  be the intersection point of the line through  $A$  perpendicular to  $AB$  and the line through  $C$  parallel to  $MD$ . Let  $AB$  intersect  $CE$  and  $DM$  at  $P$  and  $Q$ , respectively. Prove that  $Q$  is the midpoint of  $BP$ .

**334. 3 (Nguyễn Duy Liên)** Find the smallest possible odd natural number  $n$  such that  $n^2$  can be expressed as the sum of an odd number of consecutive perfect squares.

**334. 4 (Phạm Việt Hải)** Find all positive numbers  $a, b, c, d$  such that

$$\frac{a^2}{b+c} + \frac{b^2}{c+d} + \frac{c^2}{d+a} + \frac{d^2}{a+b} = 1 \quad \text{and}$$

$$a^2 + b^2 + c^2 + d^2 \geq 1.$$

**334. 5 (Đào Quốc Dũng)** The incircle of triangle  $ABC$  (incenter  $I$ ) touches the sides  $BC, CA$ , and  $AB$  respectively at  $D, E, F$ . The line through  $A$  perpendicular to  $IA$  intersects lines  $DE, DF$  at  $M, N$ , respectively; the line through  $B$  perpendicular to  $IB$  intersect  $EF, ED$  at  $P, Q$ , respectively; the line through  $C$  perpendicular to  $IC$  intersect lines  $FD, FE$  at  $S, T$ , respectively. Prove the inequality

$$MN + PQ + ST \geq AB + BC + CA.$$

**334. 6 (Vũ Hữu Bình)** Let  $ABC$  be a right isosceles triangle with  $A = 90^\circ$ . Find the locus of points  $M$  such that  $MB^2 - MC^2 = 2MA^2$ .

**334. 7 (Trần Tuấn Anh)** We are given  $n$  distinct positive numbers,  $n \geq 4$ . Prove that it is possible to choose at least two numbers such that their sums and differences do not coincide with any  $n - 2$  others of the given numbers.

**335. 1 (Vũ Tiến Việt)** Prove that for all triangles  $ABC$

$$\cos A + \cos B + \cos C \leq 1 + \frac{1}{6} \left( \cos^2 \frac{A-B}{2} + \cos^2 \frac{B-C}{2} + \cos^2 \frac{C-A}{2} \right).$$

**335. 2 (Phan Đức Tuấn)** In triangle  $ABC$ , let  $BC = a, CA = b, AB = c$  and  $F$  be its area. Suppose that  $M, N$ , and  $P$  are points on the sides  $BC, CA$ , and  $AB$ , respectively. Prove that

$$ab \cdot MN^2 + bc \cdot NP^2 + ca \cdot PM^2 \geq 4F^2.$$

**335. 3 (Trần Văn Xuân)** In isosceles triangle  $ABC$ ,  $\angle ABC = 120^\circ$ . Let  $D$  be the point of intersection of line  $BC$  and the tangent to the circumcircle of triangle  $ABC$  at  $A$ . A line through  $D$  and the circumcenter  $O$  intersects  $AB$  and  $AC$  at  $E$  and  $F$ , respectively. Let  $M$  and  $N$  be the midpoints of  $AB$  and  $AC$ . Show that  $AO, MF$  and  $NE$  are concurrent.

**336. 1 (Nguyễn Hòa)** Solve the following system of equations

$$\begin{aligned}\frac{a}{x} - \frac{b}{z} &= c - zx, \\ \frac{b}{y} - \frac{c}{x} &= a - xy, \\ \frac{c}{z} - \frac{a}{y} &= b - yz.\end{aligned}$$

**336. 2 (Phạm Văn Thuận)** Given two positive real numbers  $a, b$  such that  $a^2 + b^2 = 1$ , prove that

$$\frac{1}{a} + \frac{1}{b} \geq 2\sqrt{2} + \left( \sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}} \right)^2.$$

**336. 3 (Nguyễn Hồng Thanh)** Let  $P$  be an arbitrary point in the interior of triangle  $ABC$ . Let  $BC = a$ ,  $CA = b$ ,  $AB = c$ . Denote by  $u$ ,  $v$  and  $w$  the distances of  $P$  to the lines  $BC$ ,  $CA$ ,  $AB$ , respectively. Determine  $P$  such that the product  $uvw$  is a maximum and calculate this maximum in terms of  $a, b, c$ .

**336. 4 (Nguyễn Lâm Tuyền)** Given the polynomial  $Q(x) = (p - 1)x^p - x - 1$  with  $p$  being an odd prime number. Prove that there exist infinitely many positive integers  $a$  such that  $Q(a)$  is divisible by  $p^p$ .

**336. 5 (Hoàng Minh Dũng)** Prove that in any triangle  $ABC$  the following inequalities hold:

$$\text{a) } \cos A + \cos B + \cos C + \cot A + \cot B + \cot C \geq \frac{3}{2} + \sqrt{3};$$

$$\text{b) } \sqrt{3}(\cos A + \cos B + \cos C) + \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} \geq \frac{9\sqrt{3}}{2}.$$

**337. 1 (Nguyễn Thị Loan)** Given four real numbers  $a, b, c, d$  such that  $4a^2 + b^2 = 2$  and  $c + d = 4$ , determine the maximum value of the expression  $f = 2ac + bd + cd$ .

**337. 2 (Vũ Anh Nam)** In triangle  $ABC$ , let  $D$  be the intersection point of the internal angle bisectors  $BM$  and  $CN$ ,  $M$  on  $AC$  and  $N$  on  $AB$ . Prove that  $\angle BAC = 90^\circ$  if and only if  $2BD \cdot CD = BM \cdot CN$ .

**337. 3 (Trần Tuấn Anh)** Determine the maximum value of the expression  $f = (x - y)(y - z)(z - x)(x + y + z)$ , where  $x, y, z$  lie in the interval  $[0, 1]$ .

**337. 4 (Hàn Ngọc Đức)** Let  $n, n \geq 2$ , be a natural number,  $a, b$  be positive real numbers such that  $a < b$ . Suppose that  $x_1, x_2, \dots, x_n$  are  $n$  real numbers in the interval  $[a, b]$ . Find the maximum value of the sum

$$\sum_{1 \leq i < j \leq n} (x_i - x_j)^2.$$

**337. 5 (Lê Hoài Bắc)** A line through the incenter of a triangle  $ABC$  intersects sides  $AB$  and  $AC$  at  $M$  and  $N$ , respectively. Show that

$$\frac{MB \cdot NC}{MA \cdot NA} \leq \frac{BC^2}{4AB \cdot AC}.$$

**338. 1 (Phạm Thịnh)** Show that if  $a, b, c, d, p, q$  are positive real numbers with  $p \geq q$  then the following inequality holds:

$$\frac{a}{pb + qc} + \frac{b}{pc + qd} + \frac{c}{pd + qa} + \frac{d}{pa + qb} \geq \frac{4}{p + q}.$$

Is the inequality still true if  $p < q$ ?

**338. 2 (Trần Quang Vinh)** Determine all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfying the condition  $f(x^2 + f(y)) = y + xf(x)$  for all real numbers  $x, y$ .

**338. 3 (Trần Việt Anh)** Determine the smallest possible positive integer  $n$  such that there exists a polynomial  $p(x)$  of degree  $n$  with integer coefficients satisfying the conditions

- a)  $p(0) = 1, p(1) = 1$ ;
- b)  $p(m)$  divided by 2003 leaves remainders 0 or 1 for all integers  $m > 0$ .

**338. 4 (Hoàng Trọng Hảo)** The Fibonacci sequence  $(F_n), n = 1, 2, \dots$ , is defined by  $F_1 = F_2 = 1, F_{n+1} = F_n + F_{n-1}$  for  $n = 2, 3, 4, \dots$ . Show that if  $a \neq F_{n+1}/F_n$  for all  $n = 1, 2, 3, \dots$  then the sequence  $(x_n)$ , where

$$x_1 = a, \quad x_{n+1} = \frac{1}{1 + x_n}, \quad n = 1, 2, \dots$$

is defined and has a finite limit when  $n$  tends to infinity. Determine the limit.

**339. 1 (Ngô Văn Khương)** Given five positive real numbers  $a, b, c, d, e$  such that  $a^2 + b^2 + c^2 + d^2 + e^2 \leq 1$ , prove that

$$\frac{1}{1 + ab} + \frac{1}{1 + bc} + \frac{1}{1 + cd} + \frac{1}{1 + de} + \frac{1}{1 + ea} \geq \frac{25}{6}.$$

**339. 2 (Lê Chu Biên)** Suppose that  $ABCD$  is a rectangle. The line perpendicular to  $AC$  at  $C$  intersects lines  $AB, AD$  respectively at  $E, F$ . Prove the identity  $BE\sqrt{CF} + DF\sqrt{CE} = AC\sqrt{EF}$ .

**339. 3 (Trần Hồng Sơn)** Let  $I$  be the incenter of triangle  $ABC$  and let  $m_a, m_b, m_c$  be the lengths of the medians from vertices  $A, B$  and  $C$ , respectively. Prove that

$$\frac{IA^2}{m_a^2} + \frac{IB^2}{m_b^2} + \frac{IC^2}{m_c^2} \leq \frac{3}{4}.$$

**339. 4 (Quách Văn Giang)** Given three positive real numbers  $a, b, c$  such that  $ab + bc + ca = 1$ . Prove that the minimum value of the expression  $x^2 + ry^2 + tz^2$  is  $2m$ , where  $m$  is the root of the cubic equation  $2x^3 + (r + s + 1)x^2 - rs = 0$  in the interval  $(0, \sqrt{rs})$ . Find all primes  $r, s$  such that  $2m$  is rational.

**339. 5 (Nguyễn Trường Phong)** The sequence  $(x_n)$  is defined by

$$x_n = a_n^{a_n}, \quad \text{where } a_n = \frac{(2n)!}{(n!)^2 \cdot 2^{2n}}, \quad \text{for } n = 1, 2, 3, \dots$$

Prove that the sequence  $(x_n)$  has a limit when  $n$  tends to infinity and determine the limit.

**339. 6 (Huỳnh Tấn Châu)** Let  $a$  be a real number,  $a \in (0, 1)$ . Determine all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  that are continuous at  $x = 0$  and satisfy the equation

$$f(x) - 2f(ax) + f(a^2x) = x^2$$

for all real  $x$ .

**339. 7 (Nguyễn Xuân Hùng)** In the plane, given a circle with center  $O$  and radius  $r$ . Let  $P$  be a fixed point inside the circle such that  $OP = d > 0$ . The chords  $AB$  and  $CD$  through  $P$  make a fixed angle  $\alpha$ , ( $0^\circ < \alpha \leq 90^\circ$ ). Find the maximum and minimum value of the sum  $AB + CD$  when both  $AB$  and  $CD$  vary, and determine the position of the two chords.

**340. 1 (Phạm Hoàng Hà)** Find the maximum value of the expression

$$\frac{x+y}{1+z} + \frac{y+z}{1+x} + \frac{z+x}{1+y},$$

where  $x, y, z$  are real numbers in the interval  $[\frac{1}{2}, 1]$ .

**340. 2 (Nguyễn Quỳnh)** Let  $M$  be a point interior to triangle  $ABC$ , let  $AM$  intersect  $BC$  at  $E$ , let  $CM$  meet  $AB$  at  $F$ . Suppose that  $N$  is the reflection of  $B$  across the midpoint of  $EF$ . Prove that the line  $MN$  has a fixed point when  $M$  moves in the triangle  $ABC$ .

**340.3 (Trần Tuấn Anh)** Let  $a, b, c$  be the side lengths of a triangle, and  $F$  its area, prove that  $F \leq \frac{\sqrt{3}}{4}(abc)^{2/3}$ , and determine equality cases.

**340.4 (Hàn Ngọc Đức)** Given non-negative integers  $n, k$ ,  $n > 1$  and let  $\{a_1, a_2, \dots, a_n\}$  be the  $n$  real numbers, prove that

$$\sum_{i=1}^n \sum_{j=1}^n \frac{a_i a_j}{\binom{k+2}{k+i+j}} \geq 0.$$

**340.5 (Trần Minh Hiền)** Does there exist a function  $f : \mathbb{R}^* \rightarrow \mathbb{R}^*$  such that

$$f^2(x) \geq f(x+y)(f(x)+y)$$

for all positive real numbers  $x, y$ ?

**341.1 (Trần Tuyết Thanh)** Find all integers  $x, y, z, t$  such that

$$x^y + y^z + z^t = x^{2005}.$$

**341.2 (Nguyễn Hữu Bằng)** Solve the equation

$$(x^2 - 12x - 64)(x^2 + 30x + 125) + 8000 = 0.$$

**341.3 (Đoàn Quốc Việt)** Given an equilateral triangle  $ABC$ , let  $D$  be the reflection of  $B$  across the line  $AC$ . A line through point  $B$  intersects the lines  $AD, CD$  at  $M$  and  $N$  respectively,  $E$  is the point of intersection of  $AN$  and  $CM$ . Prove that  $A, C, D$ , and  $E$  are concyclic.

**341.4 (Nguyễn Thanh Nhân)** Prove that for every positive integer  $n > 2$ , there exist  $n$  distinct positive integers such that the sum of these numbers is equal to their least common multiple and is equal to  $n!$ .

**341.5 (Nguyễn Vũ Lương)** Prove that if  $x, y, z > 0$  then

$$\text{a) } \sqrt{\frac{x}{y+2z}} + \sqrt{\frac{y}{x+2z}} + 2\sqrt{\frac{z}{x+y+z}} > 2,$$

$$\text{b) } \sqrt[3]{\frac{x}{y+2z}} + \sqrt[3]{\frac{y}{x+2z}} + 2\sqrt[3]{\frac{z}{x+y+z}} > 2.$$

**342.1 (Trần Văn Hình)** Let  $ABC$  be an isosceles triangle with  $\angle ABC = \angle ACB = 36^\circ$ . Point  $N$  is chosen on the angle bisector of  $\angle ABC$  such that  $\angle BCN = 12^\circ$ . Compare the length of  $CN$  and  $CA$ .

**342. 2 (Cù Huy Toàn)** Find integers  $(x, y)$  such that

$$5x^2 + 4y^2 + 5 = (x^2 + y^2 + 1)^2.$$

**342. 3 (Trần Tuấn Anh)** Show that if  $a \geq 0$ , then

$$\sqrt{9+a} \geq \sqrt{a} + \frac{2\sqrt{2}}{\sqrt{1+a}}.$$

When does the equality hold?

**342. 4 (Nguyễn Minh Hà)** Given an isosceles triangle  $ABC$  with  $AB = AC$  and  $\angle BAC = 80^\circ$ . Point  $M$  is interior to the triangle such that  $\angle MAC = 20^\circ$  and  $\angle MCA = 30^\circ$ . Calculate  $\angle MBC$ .

**342. 5 (Bùi Văn Chi)** Let  $(\omega)$  be a circle. Suppose that three points  $A, B$ , and  $C$  on the circle are not diametrically symmetric, and  $AB \neq BC$ . A line passing through  $A$  perpendicular to  $OB$  intersects  $CB$  at  $N$ . Let  $M$  be the midpoint of  $AB$ , and  $D$  be the second intersection of  $BM$  and the circle. Suppose that  $OE$  is the diameter of the circle through points  $B, D$  and the center of the circle. Prove that  $A, C, E$  are collinear.

**342. 6 (Nguyễn Trọng Quân)** Let  $r, R$  be the inradius, circumradius of a triangle  $ABC$ , respectively. Prove that

$$\cos A \cos B \cos C \leq \left(\frac{r}{R\sqrt{2}}\right)^2.$$

**342. 7 (Phạm Ngọc Bội)** Let  $S$  be a set of 2005 positive numbers  $a_1, a_2, \dots, a_{2005}$ . Let  $T_i$  be the non-empty subset of  $S$ ,  $s_i$  be the sum of the numbers belonging to  $T_i$ . Prove that the set of numbers  $s_i$  can be partitioned into 2005 non-empty disjoint subsets so that the ratio of two arbitrary numbers in a subset does not exceed 2.

**342. 8 (Đỗ Thanh Sơn)** Suppose that  $a, b, c, d$  are positive real numbers such that  $(bc - ad)^2 = 3(ac + bd)^2$ . Prove that

$$\sqrt{(a-c)^2 + (b-d)^2} \geq \frac{1}{2}(\sqrt{a^2 + b^2} + \sqrt{c^2 + d^2}).$$

**342. 9 (Trần Văn Tân)** The sequence  $(x_n)$  ( $n = 1, 2, \dots$ ) is defined by  $x_1 = 1$ , and

$$x_{n+1} = \sqrt{x_n(x_n + 1)(x_n + 2)(x_n + 3) + 1}, \text{ for } n = 1, 2, \dots$$

Let

$$y_n = \frac{1}{x_1 + 2} + \frac{1}{x_2 + 2} + \dots + \frac{1}{x_n + 2}, \quad (n = 1, 2, \dots)$$

Find  $\lim_{n \rightarrow \infty} y_n$ .

**343.1 (Vĩnh Linh)** Triangle  $ABC$  has  $\angle BAC = 55^\circ$ ,  $\angle ABC = 115^\circ$ . A point  $P$  is chosen on the internal angle bisector of angle  $ACB$  such that  $\angle PAC = 25^\circ$ . With proof, find  $\angle BPC$ .

**343.2 (Phạm Hoàng Hà)** Find all natural numbers  $x, y, z$  such that  $x^3 + y^3 = 2z^3$ , and  $x + y + z$  is a prime.

**343.3 (Cao Xuân Nam)** Let  $a, b > 1$  be real numbers such that  $a + b \leq 4$ , find the minimum value of the expression

$$F = \frac{a^4}{(b-1)^3} + \frac{b^4}{(a-1)^3}.$$

**343.4 (Phạm Huy Thông)** An isosceles triangle  $ABC$  has  $BC = a$ ,  $AB = AC = b$ ,  $a > b$ . The bisector  $BD$  is equal to  $b$ , prove that

$$\left(1 + \frac{a}{b}\right)\left(\frac{a}{b} - \frac{b}{a}\right) = 1.$$

**343.5 (Nguyễn Việt Hà)** A triangle  $ABC$  has internal angle bisectors  $AP$ ,  $BQ$ , and  $CR$ . Suppose that  $\angle PQR = 90^\circ$ , find  $\angle ABC$ ,  $\angle BCA$ , and  $\angle CAB$ .

**343.6 (Phan Thành Nam)** For each each positive number  $x$ , denote by  $a(x)$  the number of prime numbers not exceeding  $x$ . For each positive integer  $m$ , denote by  $b(m)$  the number of prime divisors of  $m$ . Prove that for each positive integer  $n$ , we have

$$a(n) + a\left(\frac{n}{2}\right) + \cdots + a\left(\frac{n}{n}\right) = b(1) + b(2) + \cdots + b(n).$$

**343.7 (Lê Thừa Thành)** Without the aid of calculators, find the measure of acute angle  $x$  if

$$\cos x = \frac{1}{\sqrt{1 + (2 + \sqrt{3} - \sqrt{2} - \sqrt{6})^2}}.$$

**343.8 (Lưu Xuân Tình)** Suppose that triangle  $ABC$  has  $a^2 = 4F \cot A$ , where  $BC = a$ , and  $F$  denotes the area of triangle  $ABC$ . Let  $O$  and  $G$  be respectively the circumradius and centroid of triangle  $ABC$ . Find the angle between  $AG$  and  $OG$ .

**343.9 (Phan Tuấn Cộng)** For a triangle  $ABC$ , find  $A$ ,  $B$ , and  $C$  such that  $\sin^2 A + \sin^2 B - \sin^2 C$  is a minimum.

**343. 10 (Nguyễn Minh Hà)** On the side of triangle  $ABC$ , equilateral triangles  $ABE$ ,  $ACF$  are outwardly constructed. Let  $G$  be the center of triangle  $ABC$ , and  $K$  the midpoint of  $EF$ . Prove that  $\triangle KGC$  is right and one of its angle is  $60^\circ$ .

**343. 11 (Nguyễn Minh Hà)** Let  $ABC$  be a triangle with internal angle bisectors  $AP$ ,  $BQ$ , and  $CR$ . Let  $M$  be any point in the plane of the triangle  $ABC$  but not on its sides. Let  $X, Y$ , and  $Z$  be reflections of  $M$  across  $AP$ ,  $BQ$ , and  $CR$ . Prove that  $AX$ ,  $BY$ ,  $CZ$  are either concurrent or pairwise parallel.

**343. 12 (Nguyễn Minh Hà)** Let  $M$  be any point in the plane of triangle  $ABC$ . Let  $H, K, L$  be the projections of  $M$  on the lines  $BC$ ,  $CA$ ,  $AB$ . Find the locus of  $M$  such that  $H, K, L$  are collinear.

**200. 1 (Phạm Ngọc Quang)** In a triangle  $ABC$ , let  $BC = a$ ,  $CA = b$ ,  $AB = c$ ,  $I$  be the incenter of the triangle. Prove that

$$a.IA^2 + b.IB^2 + c.IC^2 = abc.$$

**200. 2 (Trần Xuân Đáng)** Let  $a, b, c \in \mathbb{R}$  such that  $a + b + c = 1$ , prove that

$$15(a^3 + b^3 + c^3 + ab + bc + ca) + 9abc \geq 7.$$

**200. 3 (Đặng Hùng Thắng)** Let  $a, b, c$  be integers such that the quadratic function  $ax^2 + bx + c$  has two distinct zeros in the interval  $(0, 1)$ . Find the least value of  $a$ ,  $b$ , and  $c$ .

**200. 4 (Nguyễn Đăng Phát)** A circle is tangent to the circumcircle of a triangle  $ABC$  and also tangent to side  $AB$ ,  $AC$  at  $P$ ,  $Q$  respectively. Prove that the midpoint of  $PQ$  is the incenter of triangle  $ABC$ . With edge and compass, construct the circle tangent to sides  $AB$  and  $AC$  and to the circle  $(ABC)$ .

**200. 5 (Nguyễn Văn Mậu)** Let  $x, y, z, t \in [1, 2]$ , find the smallest positive possible  $p$  such that the inequality holds

$$\frac{y+t}{x+z} + \frac{z+t}{t+x} \leq p \left( \frac{y+z}{x+y} + \frac{x+z}{y+t} \right).$$

**200. 6 (Nguyễn Minh Hà)** Let  $a, b, c$  be real positive numbers such that  $a + b + c = \pi$ , prove that  $\sin a + \sin b + \sin c + \sin(a + b + c) \leq \sin(a + b) + \sin(b + c) + \sin(c + a)$ .

**208. 1 (Đặng Hùng Thắng)** Let  $a_1, a_2, \dots, a_n$  be the odd numbers, none of which has a prime divisors greater than 5, prove that

$$\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} < \frac{15}{8}.$$

**208. 2 (Trần Văn Vương)** Prove that if  $r$ , and  $s$  are real numbers such that  $r^3 + s^3 > 0$ , then the equation  $x^3 + 3rx - 2s = 0$  has a unique solution

$$x = \sqrt[3]{s + \sqrt{s^2 + r^3}} + \sqrt[3]{s - \sqrt{s^2 - r^3}}.$$

Using this result to solve the equations  $x^3 + x + 1 = 0$ , and  $20x^3 - 15x^2 - 1 = 0$ .

**209. 1 (Đặng Hùng Thắng)** Find integer solutions  $(x, y)$  of the equation

$$(x^2 + y)(x + y^2) = (x - y)^3.$$

**209. 2 (Trần Duy Hình)** Find all natural numbers  $n$  such that  $n^{n+1} + (n+1)^n$  is divisible by 5.

**209. 3 (Đào Trường Giang)** Given a right triangle with hypotenuse  $BC$ , the incircle of the triangle is tangent to the sides  $AB$  and  $BC$  respectively at  $P$ , and  $Q$ . A line through the incenter and the midpoint  $F$  of  $AC$  intersects side  $AB$  at  $E$ ; the line through  $P$  and  $Q$  meets the altitude  $AH$  at  $M$ . Prove that  $AM = AE$ .

**213. 1 (Hồ Quang Vinh)** Let  $a, b, c$  be positive real numbers such that  $a + b + c = 2r$ , prove that

$$\frac{ab}{r-c} + \frac{bc}{r-a} + \frac{ca}{r-b} \geq 4r.$$

**213. 2 (Phạm Văn Hùng)** Let  $ABC$  be a triangle with altitude  $AH$ , let  $M, N$  be the midpoints of  $AB$  and  $AC$ . Prove that the circumcircles of triangles  $HBM, HCN$ , and  $AMN$  has a common point  $K$ , prove that the extended  $HK$  is through the midpoint of  $MN$ .

**213. 3 (Nguyễn Minh Đức)** Given three sequences of numbers  $\{x_n\}_{n=0}^{\infty}$ ,  $\{y_n\}_{n=0}^{\infty}$ ,  $\{z_n\}_{n=0}^{\infty}$  such that  $x_0, y_0, z_0$  are positive,  $x_{n+1} = y_n + \frac{1}{z_n}$ ,  $y_{n+1} = z_n + \frac{1}{x_n}$ ,  $z_{n+1} = x_n + \frac{1}{y_n}$  for all  $n \geq 0$ . Prove that there exist positive numbers  $s$  and  $t$  such that  $s\sqrt{n} \leq x_n \leq t\sqrt{n}$  for all  $n \geq 1$ .

**216. 1 (Thới Ngọc Ánh)** Solve the equation

$$(x+2)^2 + (x+3)^3 + (x+4)^4 = 2.$$

**216. 2 (Lê Quốc Hán)** Denote by  $(O, R)$ ,  $(I, R_a)$  the circumcircle, and the excircle of angle  $A$  of triangle  $ABC$ . Prove that

$$IA \cdot IB \cdot IC = 4R \cdot R_a^2.$$

**216. 3 (Nguyễn Đễ)** Prove that if  $-1 < a < 1$  then

$$\sqrt[4]{1-a^2} + \sqrt[4]{1-a} + \sqrt[4]{1+a} < 3.$$

**216. 4 (Trần Xuân Đáng)** Let  $(x_n)$  be a sequence such that  $x_1 = 1$ ,  $(n+1)(x_{n+1} - x_n) \geq 1 + x_n$ ,  $\forall n \geq 1$ ,  $n \in \mathbb{N}$ . Prove that the sequence is not bounded.

**216. 5 (Hoàng Đức Tân)** Let  $P$  be any point interior to triangle  $ABC$ , let  $d_A, d_B, d_C$  be the distances of  $P$  to the vertices  $A, B, C$  respectively. Denote by  $p, q, r$  distances of  $P$  to the sides of the triangle. Prove that

$$d_A^2 \sin^2 A + d_B^2 \sin^2 B + d_C^2 \sin^2 C \leq 3(p^2 + q^2 + r^2).$$

**220. 1 (Trần Duy Hình)** Does there exist a triple of distinct numbers  $a, b, c$  such that

$$(a-b)^5 + (b-c)^5 + (c-a)^5 = 0.$$

**220. 2 (Phạm Ngọc Quang)** Find triples of three non-negative integers  $(x, y, z)$  such that  $3x^2 + 54 = 2y^2 + 4z^2$ ,  $5x^2 + 74 = 3y^2 + 7z^2$ , and  $x + y + z$  is a minimum.

**220. 3 (Đặng Hùng Thắng)** Given a prime number  $p$  and positive integer  $a$ ,  $a \leq p$ , suppose that  $A = \sum_{k=0}^{p-1} a^k$ . Prove that for each prime divisor  $q$  of  $A$ , we have  $q-1$  is divisible by  $p$ .

**220. 4 (Ngọc Đạm)** The bisectors of a triangle  $ABC$  meet the opposite sides at  $D, E, F$ . Prove that the necessary and sufficient condition in order for triangle  $ABC$  to be equilateral is

$$\text{Area}(DEF) = \frac{1}{4} \text{Area}(ABC).$$

**220. 5 (Phạm Hiến Bằng)** In a triangle  $ABC$ , denote by  $l_a, l_b, l_c$  the internal angle bisectors,  $m_a, m_b, m_c$  the medians, and  $h_a, h_b, h_c$  the altitudes to the sides  $a, b, c$  of the triangle. Prove that

$$\frac{m_a}{l_b + h_b} + \frac{m_b}{l_c + h_c} + \frac{m_c}{l_a + h_a} \geq \frac{3}{2}.$$

**220.6 (Nguyễn Hữu Thảo)** Solve the system of equations

$$\begin{aligned}x^2 + y^2 + xy &= 37, \\x^2 + z^2 + zx &= 28, \\y^2 + z^2 + yz &= 19.\end{aligned}$$

**221.1 (Ngô Hân)** Find the greatest possible natural number  $n$  such that 1995 is equal to the sum of  $n$  numbers  $a_1, a_2, \dots, a_n$ , where  $a_i$ , ( $i = 1, 2, \dots, n$ ) are composite numbers.

**221.2 (Trần Duy Hình)** Find integer solutions  $(x, y)$  of the equation  $x(1 + x + x^2) = 4y(y + 1)$ .

**221.3 (Hoàng Ngọc Cảnh)** Given a triangle with incenter  $I$ , let  $\ell$  be variable line passing through  $I$ . Let  $\ell$  intersect the ray  $CB$ , sides  $AC$ ,  $AB$  at  $M, N, P$  respectively. Prove that the value of

$$\frac{AB}{PA.PB} + \frac{AC}{NA.NC} - \frac{BC}{MB.MC}$$

is independent of the choice of  $\ell$ .

**221.4 (Nguyễn Đức Tấn)** Given three integers  $x, y, z$  such that  $x^4 + y^4 + z^4 = 1984$ , prove that  $p = 20^x + 11^y - 1996^z$  can not be expressed as the product of two consecutive natural numbers.

**221.5 (Nguyễn Lê Dũng)** Prove that if  $a, b, c > 0$  then

$$\frac{a^2 + b^2}{a + b} + \frac{b^2 + c^2}{b + c} + \frac{c^2 + a^2}{c + a} \leq \frac{3(a^2 + b^2 + c^2)}{a + b + c}.$$

**221.6 (Trịnh Bằng Giang)** Let  $I$  be an interior point of triangle  $ABC$ . Lines  $IA, IB, IC$  meet  $BC, CA, AB$  respectively at  $A', B', C'$ . Find the locus of  $I$  such that

$$(IAC')^2 + (IBA')^2 + (ICB')^2 = (IBC')^2 + (ICA')^2 + (IAB')^2,$$

where  $(.)$  denotes the area of the triangle.

**221.7 (Hồ Quang Vinh)** The sequences  $(a_n)_{n \in \mathbb{N}^*}$ ,  $(b_n)_{n \in \mathbb{N}^*}$  are defined as follows

$$\begin{aligned}a_n &= 1 + \frac{n(1+n)}{1+n^2} + \dots + \frac{n^n(1+n^n)}{1+n^{2n}} \\b_n &= \left( \frac{a_n}{n+1} \right)^{\frac{1}{n(n+1)}}, \forall n \in \mathbb{N}^*.\end{aligned}$$

Find  $\lim_{n \rightarrow \infty} b_n$ .

**242. 1 (Phạm Hữu Hoài)** Let  $\alpha, \beta, \gamma$  real numbers such that  $\alpha \leq \beta \leq \gamma$ ,  $\alpha < \beta$ . Let  $a, b, c \in [\alpha, \beta]$  such that  $a + b + c = \alpha + \beta + \gamma$ . Prove that

$$a^2 + b^2 + c^2 \leq \alpha^2 + \beta^2 + \gamma^2.$$

**242. 2 (Lê Văn Bảo)** Let  $p$  and  $q$  be the perimeter and area of a rectangle, prove that

$$p \geq \frac{32q}{2q + p + 2}.$$

**242. 3 (Tô Xuân Hải)** In triangle  $ABC$  with one angle exceeding  $\frac{2}{3}\pi$ , prove that

$$\tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2} \geq 4 - \sqrt{3}.$$

**230. 1 (Trần Nam Dũng)** Let  $m \in \mathbb{N}$ ,  $m \geq 2$ ,  $p \in \mathbb{R}$ ,  $0 < p < 1$ . Let  $a_1, a_2, \dots, a_m$  be real positive numbers. Put  $s = \sum_{i=1}^m a_i$ . Prove that

$$\sum_{i=1}^m \left( \frac{a_i}{s - a_i} \right)^p \geq \frac{1}{1 - p} \left( \frac{1 - p}{p} \right)^p,$$

with equality if and only if  $a_1 = a_2 = \dots = a_m$  and  $m(1 - p) = 1$ .

**241. 3 (Nguyễn Khánh Trình, Trần Xuân Đáng)** Prove that in any acute triangle  $ABC$ , we have the inequality

$$\sin A \sin B + \sin B \sin C + \sin C \sin A \leq (\cos A + \cos B + \cos C)^2.$$

**243. 2 (Ngô Đức Minh)** Solve the equation

$$\sqrt{4x^2 + 5x + 1} - 2\sqrt{x^2 - x + 1} = 9x - 3.$$

**243. 3 (Trần Nam Dũng)** Given  $2n$  real numbers  $a_1, a_2, \dots, a_n; b_1, b_2, \dots, b_n$ , suppose that  $\sum_{j=1}^n a_j \neq 0$  and  $\sum_{j=1}^n b_j \neq 0$ . Prove that the following inequality

$$\sum_{j=1}^n a_j b_j + \left\{ \left( \sum_{j=1}^n a_j^2 \right) \left( \sum_{j=1}^n b_j^2 \right) \right\}^{\frac{1}{2}} \geq \frac{2}{n} \left( \sum_{j=1}^n a_j \right) \left( \sum_{j=1}^n b_j \right),$$

with equality if and only if

$$\frac{a_i}{\sum_{j=1}^n a_j} + \frac{b_i}{\sum_{j=1}^n b_j} = \frac{2}{n}, \quad i = 1, 2, \dots, n.$$

**243. 4 (Hà Đức Vương)** Given a triangle  $ABC$ , let  $AD$  and  $AM$  be the internal angle bisector and median of the triangle respectively. The circum-circle of  $ADM$  meet  $AB$  and  $AC$  at  $E$ , and  $F$  respectively. Let  $I$  be the midpoint of  $EF$ , and  $N, P$  be the intersections of the line  $MI$  and the lines  $AB$  and  $AC$  respectively. Determine, with proof, the shape of the triangle  $ANP$ .

**243. 5 (Tô Xuân Hải)** Prove that

$$\arctan \frac{1}{5} + \arctan 2 + \arctan 3 - \arctan \frac{1}{239} = \pi.$$

**267. 4 (Vũ Ngọc Minh, Phạm Gia Vĩnh Anh)** Let  $a, b$  be real positive numbers,  $x, y, z$  be real numbers such that

$$\begin{aligned} x^2 + z^2 &= b, \\ y^2 + (a - b)y(z + x) &= 2ab^2. \end{aligned}$$

Prove that  $y(z - x) \leq (a + b)b$  with equality if and only if

$$x = \pm \frac{a\sqrt{b}}{\sqrt{a^2 + b^2}}, \quad z = \mp \frac{b\sqrt{b}}{\sqrt{a^2 + b^2}}, \quad z = \mp \sqrt{b(a^2 + b^2)}.$$

**295. 7 (Hoàng Văn Đắc)** Let  $a, b, c, d \in \mathbb{R}$  such that  $a + b + c + d = 1$ , prove that

$$(a + c)(b + d) + 2(ac + bd) \leq \frac{1}{2}.$$

**344. 1 (Vũ Hữu Chín)** Let  $ABC$  be a right isosceles triangle with hypotenuse  $BC$ . Let  $M$  be the midpoint of  $BC$ ,  $G$  be a point chosen on the side  $AB$  such that  $AG = \frac{1}{3}AB$ ,  $E$  be the foot of the perpendicular from  $M$  on  $CG$ . Let  $MG$  intersect  $AC$  at  $D$ , compare  $DE$  and  $BC$ .

**344. 2 (Hoàng Anh Tuấn)** Solve the equation

$$\frac{2 + x}{\sqrt{2} + \sqrt{2 + x}} + \frac{2 - x}{\sqrt{2} - \sqrt{2 - x}} = \sqrt{2}.$$

**344. 3 (Vũ Đức)** Solve the system of equations

$$\begin{aligned} x^2 + y^2 &= 1, \\ 3x^3 - y^3 &= \frac{1}{x + y}. \end{aligned}$$

**344. 4 (Tạ Hoàng Thông)** Let  $a, b, c$  be positive real numbers such that  $a^2 + b^2 + c^2 = 3$ , find the greatest possible constant  $\lambda$  such that

$$ab^2 + bc^2 + ca^2 \geq \lambda(ab + bc + ca)^2.$$

**344. 5 (Hàn Ngọc Đức)** Let  $X$  be any point on the side  $AB$  of the parallelogram  $ABCD$ . A line through  $X$  parallel to  $AD$  intersects  $AC$  at  $M$  and intersects  $BD$  at  $N$ ;  $XD$  meets  $AC$  at  $P$  and  $XC$  cuts  $BD$  at  $Q$ . Prove that

$$\frac{MP}{AC} + \frac{NQ}{BD} \geq \frac{1}{3}.$$

When does equality hold?

**344. 6 (Hồ Quang Vinh)** Given a triangle  $ABC$  with altitudes  $AM, BN$  and inscribed circle  $(\Gamma)$ , let  $D$  be a point on the circle such that  $D$  is distinct from  $A, B$  and  $DA$  and  $BN$  have a common point  $Q$ . The line  $DB$  intersects  $AM$  at  $P$ . Prove that the midpoint of  $PQ$  lies on a fixed line as  $D$  varies on the circle  $(\Gamma)$ .

**344. 7 (Lưu Bá Thắng)** Let  $p$  be an odd prime number, prove that

$$\sum_{j=0}^p \binom{j}{p} \binom{j}{p+j} - (2^p + 1)$$

is divisible by  $p^2$ .

**344. 8 (Trần Nguyễn An)** Let  $\{f_n(x)\}$ ,  $(n = 0, 1, 2, \dots)$  be a sequence of functions defined on  $[0, 1]$  such that

$$f_0(x) = 0, \text{ and } f_{n+1}(x) = f_n(x) + \frac{1}{2}(x - (f_n(x))^2) \text{ for } n = 0, 1, 2, \dots$$

Prove that  $\frac{nx}{2 + n\sqrt{x}} \leq f_n(x) \leq \sqrt{x}$ , for  $n \in \mathbb{N}, x \in [0, 1]$ .

**344. 9 (Trần Nguyễn Bình)** Given a polynomial  $p(x) = x^2 - 1$ , find the number of distinct zeros of the equation

$$p(p(\dots(p(x))\dots)) = 0,$$

where there exist 2006 notations of  $p$  inside the equation.

**344. 10 (Nguyễn Minh Hà)** Let  $ABCDEF$  be a convex inscribable hexagon. The diagonal  $BF$  meets  $AE, AC$  respectively at  $M, N$ ; diagonal  $BD$  intersects  $CA, CE$  at  $P, Q$  in that order, diagonal  $DF$  cuts  $EC, EA$  at  $R, S$  respectively. Prove that  $MQ, NR$ , and  $PS$  are concurrent.

**344. 11 (Vietnam 1991)** Let  $A, B, C$  be angles of a triangle, find the minimum of

$$(1 + \cos^2 A)(1 + \cos^2 B)(1 + \cos^2 C).$$

**344. 12 (Vietnam 1991)** Let  $x_1, x_2, \dots, x_n$  be real numbers in the interval  $[-1; 1]$ , and  $x_1 + x_2 + \dots + x_n = n - 3$ , prove that

$$x_1^2 + x_2^2 + \dots + x_{n-1}^2 + x_n^2 \leq n - 1.$$

**345. 1 (Trần Tuấn Anh)** Let  $x, y$  be real numbers in the interval  $[0, \frac{1}{\sqrt{2}}]$ , find the maximum of

$$p = \frac{x}{1 + y^2} + \frac{y}{1 + x^2}.$$

**345. 2 (Cù Huy Toàn)** Prove that

$$\frac{3\sqrt{3}}{4} \leq \frac{yz}{x(1 + yz)} + \frac{zx}{y(1 + zx)} + \frac{xy}{z(1 + xy)} \leq \frac{1}{4}(x + y + z),$$

where  $x, y, z$  are positive real numbers such that  $x + y + z = xyz$ .

**345. 3 (Hoàng Hải Dương)** Points  $E$ , and  $D$  are chosen on the sides  $AB$ ,  $AC$  of triangle  $ABC$  such that  $AE/EB = CD/DA$ . Let  $M$  be the intersection of  $BD$  and  $CE$ . Locate  $E$  and  $D$  such that the area of triangle  $BMC$  is a maximum, and determine the area in terms of triangle  $ABC$ .

**345. 4 (Hoàng Trọng Hào)** Find all  $x$  such that the following is an integer.

$$\frac{\sqrt{x}}{x\sqrt{x} - 3\sqrt{x} + 3}.$$

**345. 5 (Lê Hoài Bắc)** Let  $ABC$  be a triangle inscribable in circle  $(\Gamma)$ . Let the bisector of  $\angle BAC$  meet the circle at  $A$  and  $D$ , the circle with center  $D$ , diameter  $AD$  meets the line  $AB$  at  $B$  and  $Q$ , intersects the line  $AC$  at  $C$  and  $O$ . Prove that  $AO$  is perpendicular to  $PQ$ .

**345. 6 (Nguyễn Trọng Tuấn)** Determine all the non-empty subsets  $A, B, C$  of  $\mathbb{N}$  such that

- i)  $A \cap B = B \cap C = C \cap A = \emptyset$ ;
- ii)  $A \cup B \cup C = \mathbb{N}$ ;
- iii) For all  $a \in A, b \in B, c \in C$  then  $a + c \in A, b + c \in B$ , and  $a + b \in C$ .

**345. 7 (Nguyễn Trọng Hiệp)** Find all the functions  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  satisfying the following conditions

- i)  $f(f(m - n)) = f(m^2) + f(n) - 2n \cdot f(m)$  for all  $m, n \in \mathbb{Z}$ ;
- ii)  $f(1) > 0$ .

**345. 8 (Nguyễn Đễ)** Let  $AM, BN, CP$  be the medians of triangle  $ABC$ . Prove that if the radius of the incircles of triangles  $BCN, CAP$ , and  $ABM$  are equal in length, then  $ABC$  is an equilateral triangle.

**346. 1 (Đỗ Bá Chủ)** Determine, with proof, the minimum of

$$(x^2 + 1)\sqrt{x^2 + 1} - x\sqrt{x^4 + 2x^2 + 5} + (x - 1)^2.$$

**346. 2 (Hoàng Hùng)** The quadrilateral  $ABCD$  is inscribed in the circle  $(O)$  and  $AB$  intersects  $CD$  at some point, let  $I$  be the point of intersection of the two diagonals. Let  $M$  and  $N$  be the midpoints of  $BC$  and  $CD$ . Prove that if  $NI$  is perpendicular to  $AB$  then  $MI$  is perpendicular to  $AD$ .

**346. 3 (Trần Quốc Hoàn)** Given six positive integers  $a, b, c, d, e$ , and  $f$  such that  $abc = def$ , prove that

$$a(b^2 + c^2 + d(e^2 + f^2))$$

is a whole number.

**346. 4 (Bùi Đình Thân)** Given quadratic trinomials of the form  $f(x) = ax^2 + bx + c$ , where  $a, b, c$  are integers and  $a > 0$ , has two distinct roots in the interval  $(0, 1)$ . Find all the quadratic trinomials and determine the one with the smallest possible leading coefficient.

**346. 5 (Phạm Kim Hùng)** Prove that

$$xy + yz + zx \geq 8(x^2 + y^2 + z^2)(x^2y^2 + y^2z^2 + z^2x^2),$$

where  $x, y, z$  are non-negative numbers such that  $x + y + z = 1$ .

**346. 6 (Lam Sơn, Thanh Hoa)** Let  $x, y, z$  be real numbers greater than 2 such that  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ , prove that

$$(x - 2)(y - 2)(z - 2) \leq 1.$$

**346. 7 (Huỳnh Duy Thuỷ)** Given a polynomial  $f(x) = mx^2 + (n - p)x + m + n + p$  with  $m, n, p$  being real numbers such that  $(m + n)(m + n + p) \leq 0$ , prove that

$$\frac{n^2 + p^2}{2} \geq 2m(m + n + p) + np.$$

**346. 8 (Vũ Thái Lộc)** The incircle  $(I)$  of a triangle  $A_1A_2A_3$  with radius  $r$  touches the sides  $A_2A_3, A_3A_1, A_1A_2$  respectively at  $M_1, M_2, M_3$ . Let  $(I_i)$  be the circle touching the sides  $A_iA_j, A_iA_k$  and externally touching  $(I)$  ( $i, j, k \in \{1, 2, 3\}, i \neq j \neq k \neq i$ ). Let  $K_1, K_2, K_3$  be the points of tangency of  $(I_1)$  with  $A_1A_2$ , of  $(I_2)$  with  $A_2A_3$ , of  $(I_3)$  with  $A_3A_1$  respectively. Let  $A_iA_i = a_i, A_iK_i = b_i, (i = 1, 2, 3)$ , prove that

$$\frac{1}{r} \sum_{i=1}^3 (a_i + b_i) \geq 2 + \sqrt{3}.$$

When does equality hold?

... to be continued