

$$\begin{aligned}
\cos \alpha + \cos \beta + \cos \gamma &= 1 + 4 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2} = \frac{R+r}{R}, \\
\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma &= \frac{6R^2 + 4Rr + r^2 - s^2}{2R^2}, \\
\cos \beta \cos \gamma + \cos \gamma \cos \alpha + \cos \alpha \cos \beta &= \frac{s^2 - 4R^2 + r^2}{4R^2}, \\
\cos \alpha \cos \beta \cos \gamma &= \frac{s^2 - (2R+r)^2}{4R^2}, \\
(\cos \beta + \cos \gamma)(\cos \gamma + \cos \alpha)(\cos \alpha + \cos \beta) &= \frac{r(s^2 + 2Rr + r^2)}{4R^3}, \\
\cos^3 \alpha + \cos^3 \beta + \cos^3 \gamma &= \frac{(2R+r)^3 - 3rs^2}{4R^3} - 1, \\
\frac{\cos \beta + \cos \gamma}{\cos \alpha} + \frac{\cos \gamma + \cos \alpha}{\cos \beta} + \frac{\cos \alpha + \cos \beta}{\cos \gamma} &= \frac{(R+r)(s^2 - 4R^2 + r^2)}{R[s^2 - (2R+r)^2]} - 3,
\end{aligned}$$