$$\sin^2\frac{\alpha}{2}+\sin^2\frac{\beta}{2}+\sin^2\frac{\gamma}{2}=\frac{3-(\cos\alpha+\cos\beta+\cos\gamma)}{2}=\frac{2R-r}{2R},$$
$$\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}=\frac{1}{4}(\cos\alpha+\cos\beta+\cos\gamma-1)=\frac{r}{4R}=\frac{(s-a)(s-b)(s-c)}{abc},$$
$$\sin^4\frac{\alpha}{2}+\sin^4\frac{\beta}{2}+\sin^4\frac{\gamma}{2}=\frac{8R^2+r^2-s^2}{8R^2},$$
$$\sin^2\frac{\beta}{2}\sin^2\frac{\gamma}{2}+\sin^2\frac{\gamma}{2}\sin^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}=\frac{s^2-8Rr+r^2}{16R^2},$$
$$\cos^2\frac{\alpha}{2}+\cos^2\frac{\beta}{2}+\cos^2\frac{\gamma}{2}=\frac{4R+r}{2R},$$
$$\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}=\frac{s}{4R}=\frac{\Delta}{4Rr}=\frac{abc}{16R^2r},$$
$$\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}+\cos^2\frac{\gamma}{2}\cos^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}=\frac{s^2+(4R+r)^2}{16R^2},$$
$$\tan^2\frac{\alpha}{2}+\tan^2\frac{\beta}{2}+\tan^2\frac{\gamma}{2}=\frac{(4R+r)^2-2s^2}{s^2},$$
$$\tan\frac{\beta}{2}\tan\frac{\gamma}{2}=\frac{s-a}{s},\qquad \tan\frac{\gamma}{2}\tan\frac{\alpha}{2}=\frac{s-b}{s},\qquad \tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{s-c}{s},$$
$$\tan\frac{\beta}{2}\tan\frac{\gamma}{2}+\tan\frac{\gamma}{2}\tan\frac{\alpha}{2}+\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=1,$$
$$\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}=\frac{r}{s},$$
$$\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}+\cot\frac{\gamma}{2}=\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}=\frac{s}{r},$$
$$\sin 2\alpha+\sin 2\beta+\sin 2\gamma=4\sin\alpha\sin\beta\sin\gamma=\frac{a\cos\alpha+b\cos\beta+c\cos\gamma}{R}=\frac{2rs}{R^2},$$
$$\sin 2\alpha\sin 2\beta\sin 2\gamma=8\sin\alpha\sin\beta\sin\gamma\cos\alpha\cos\beta\cos\gamma=\frac{rs\,[s^2-(2R+r)^2]}{R^4},$$
$$\cos 2\alpha+\cos 2\beta+\cos 2\gamma=\frac{3R^2+4Rr+r^2-s^2}{R^2},$$
$$\sin(\beta-\gamma)+\sin(\gamma-\alpha)+\sin(\alpha-\beta)=-4\sin\left(\frac{\beta-\gamma}{2}\right)\sin\left(\frac{\gamma-\alpha}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right),$$
$$\cos\left(\frac{\beta-\gamma}{2}\right)\cos\left(\frac{\gamma-\alpha}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{s^2+2Rr+r^2}{8R^2}.$$

Weitere Formeln gewinnt man durch zyklische Vertauschung der Winkel (α,β,γ).
Nachfolgend noch einige Ungleichungen:

$$\cos\beta+\cos\gamma\le 2\sin\frac{\alpha}{2},$$
$$\cos\alpha+\cos\beta+\cos\gamma\le\sin\frac{\alpha}{2}+\sin\frac{\beta}{2}+\sin\frac{\gamma}{2}.$$

(Wird fortgesetzt.)