

## T.1.2 Längen- und Flächenbeziehungen

$$\sigma_1 \equiv a + b + c = 2s,$$

$$\sigma_2 \equiv bc + ca + ab = s^2 + 4Rr + r^2 = 2\Delta \left( \frac{1}{\sin \alpha} + \frac{1}{\sin \beta} + \frac{1}{\sin \gamma} \right) = 2R(h_a + h_b + h_c),$$

$$\sigma_3 \equiv abc = 4Rrs = 4R\Delta,$$

$$s_2 \equiv a^2 + b^2 + c^2 = 2(s^2 - 4Rr - r^2) = 2R(a \sin \alpha + b \sin \beta + c \sin \gamma),$$

$$s_3 \equiv a^3 + b^3 + c^3 = 2s(s^2 - 6Rr - 3r^2),$$

$$(a + b)(b + c)(c + a) = 2s(s^2 + 2Rr + r^2),$$

$$(bc)^2 + (ca)^2 + (ab)^2 = 8\Delta^2 + \frac{1}{2}(a^4 + b^4 + c^4) = \frac{4\Delta^2}{\sin^2 \omega},$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{s^2 + 4Rr + r^2}{4Rrs},$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \left( \frac{s^2 - 4Rr + r^2}{4Rrs} \right)^2 - \frac{1}{Rr},$$

$$\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab} = \frac{2s}{abc} = \frac{1}{2Rr},$$

$$\frac{b + c}{a} + \frac{c + a}{b} + \frac{a + b}{c} = \frac{s^2 - 2Rr + r^2}{2Rr},$$

$$\frac{a}{s - a} + \frac{b}{s - b} + \frac{c}{s - c} = \frac{4R - 2r}{r},$$

$$\frac{a^2}{s - a} + \frac{b^2}{s - b} + \frac{c^2}{s - c} = \frac{4s(R - r)}{r},$$

$$\frac{a}{(s - b)(s - c)} + \frac{b}{(s - c)(s - a)} + \frac{c}{(s - a)(s - b)} = \frac{2(4R + r)}{rs},$$

$$\frac{a^2}{(s - b)(s - c)} + \frac{b^2}{(s - c)(s - a)} + \frac{c^2}{(s - a)(s - b)} = \frac{4(R + r)}{r},$$

$$OH^2 = 9R^2 - (a^2 + b^2 + c^2) = R^2(1 - 8 \cos \alpha \cos \beta \cos \gamma),$$

$$OI^2 = R^2 - 2Rr,$$

$$IH^2 = 2r^2 - 4R^2 \cos \alpha \cos \beta \cos \gamma = 4R^2 + 4Rr + 3r^2 - s^2,$$

$$GI^2 = \frac{2}{3}OI^2 + \frac{1}{3}IH^2 + \frac{2}{9}OH^2 = \frac{1}{9}(s^2 - 16Rr + 5r^2),$$

$$GH = 2OG,$$

$$\begin{aligned} [OIH] &= 2R^2 \sin \frac{\beta - \gamma}{2} \sin \frac{\alpha - \gamma}{2} \sin \frac{\alpha - \beta}{2} \\ &= \frac{(b - c)(a - c)(a - b)}{8r} \quad (a > b > c). \end{aligned}$$