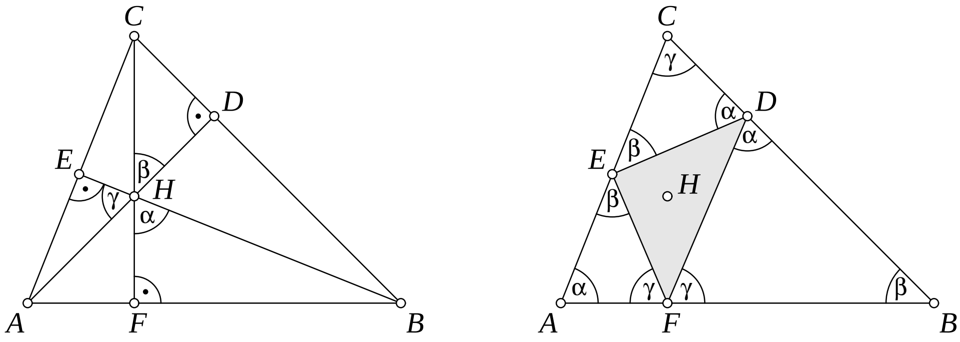
T.1.6 Formeln – Höhenschnittpunkt



$$\measuredangle AFC = \measuredangle BFC = \measuredangle BDA = \measuredangle CDA = \measuredangle CEB = \measuredangle AEB = 90^\circ,$$

$$h_a = AD = \frac{2rs}{a} = \frac{bc}{2R} = 2R\sin\beta\sin\gamma, \qquad h_b = BE = \frac{2rs}{b} = \frac{ca}{2R} = 2R\sin\gamma\sin\alpha,$$

$$h_c = CF = \frac{2rs}{c} = \frac{ab}{2R} = 2R\sin\alpha\sin\beta,$$

$$\frac{2}{h_a} = \frac{1}{r_b} + \frac{1}{r_c}, \qquad \frac{2}{h_b} = \frac{1}{r_c} + \frac{1}{r_a}, \qquad \frac{2}{h_c} = \frac{1}{r_a} + \frac{1}{r_b},$$

$$\frac{a}{h_a} = \cot\beta + \cot\gamma, \qquad \frac{b}{h_b} = \cot\gamma + \cot\alpha, \qquad \frac{c}{h_c} = \cot\alpha + \cot\beta,$$

$$\frac{h_b h_c}{h_a} = \frac{2a\Delta}{bc} = \frac{a^2}{2R} = a\sin\alpha, \qquad \frac{h_c h_a}{h_b} = \frac{2b\Delta}{ca} = \frac{b^2}{2R} = b\sin\beta,$$

$$\frac{h_a h_b}{h_c} = \frac{2c\Delta}{ab} = \frac{c^2}{2R} = c\sin\gamma,$$

$$h_a + h_b + h_c = \frac{bc + ca + ab}{2R} = 2\Delta\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right),$$

$$\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r} = \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c},$$

$$h_b h_c + h_c h_a + h_a h_b = \frac{2rs^2}{R} = \frac{2\Delta^2}{Rr},$$

$$\frac{h_b h_c}{h_a} + \frac{h_c h_a}{h_b} + \frac{h_a h_b}{h_c} = a\sin\alpha + b\sin\beta + c\sin\gamma = a^2 + b^2 + c^2,$$

$$h_a h_b h_c = \frac{2\Delta^2}{R},$$

$$AF = \frac{b^2 + c^2 - a^2}{2c} = b\cos\alpha = h_c\cot\alpha, \qquad FB = \frac{c^2 + a^2 - b^2}{2c} = a\cos\beta = h_c\cot\beta,$$

$$BD = \frac{c^2 + a^2 - b^2}{2a} = c\cos\beta = h_a\cot\beta, \qquad DC = \frac{a^2 + b^2 - c^2}{2a} = b\cos\gamma = h_a\cot\gamma,$$

$$CE = \frac{a^2 + b^2 - c^2}{2b} = a\cos\gamma = h_b\cot\gamma, \qquad EA = \frac{b^2 + c^2 - a^2}{2b} = c\cos\alpha = h_b\cot\alpha,$$