T.5.5 Vektorielle Identitäten

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b}), \quad \text{(BAC-CAB-Identität)}$$

$$|\boldsymbol{a} + \boldsymbol{b}|^2 + |\boldsymbol{a} - \boldsymbol{b}|^2 = 2\,|\boldsymbol{a}|^2 + 2\,|\boldsymbol{b}|^2, \tag{T.62}$$

$$|\boldsymbol{a} - \boldsymbol{b} - \boldsymbol{c}|^2 + |\boldsymbol{b} - \boldsymbol{c} - \boldsymbol{a}|^2 + |\boldsymbol{c} - \boldsymbol{a} - \boldsymbol{b}|^2 = |\boldsymbol{a} - \boldsymbol{b}|^2 + |\boldsymbol{b} - \boldsymbol{c}|^2 + |\boldsymbol{c} - \boldsymbol{a}|^2 + (a^2 + b^2 + c^2), \tag{T.63}$$

$$|\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}|^2 - |\boldsymbol{b} + \boldsymbol{c}|^2 - |\boldsymbol{c} + \boldsymbol{a}|^2 - |\boldsymbol{a} + \boldsymbol{b}|^2 + |\boldsymbol{a}|^2 + |\boldsymbol{b}|^2 + |\boldsymbol{c}|^2 = 0. \tag{T.64}$$

Für beliebige vier Punkte $A, B, C, D \in \mathbb{R}^n$ gilt:

$$\overrightarrow{AB} \cdot \overrightarrow{CD} + \overrightarrow{AC} \cdot \overrightarrow{DB} + \overrightarrow{AD} \cdot \overrightarrow{BC} = 0. \tag{T.65}$$