T.5.6 Hypergeometrische Identitäten

Die folgenden Summen beinhalten hauptsächlich Binomialkoeffizienten.

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n, \tag{T.64}$$

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}, \tag{T.65}$$

$$\sum_{k=0}^{n} (-1)^{n+k}\, 2^k \binom{n}{k} = 1, \tag{T.66}$$

$$\sum_{k=0}^{n} x^k \binom{n}{k} = (x+1)^n, \tag{T.67}$$

$$\sum_{k=0}^{n} (-1)^k (x-k)^n \binom{n}{k} = n!, \quad (\text{Euler}) \tag{T.68}$$

$$\sum_{k=0}^{n} (k+r)^{k-1}(s-k)^{n-k} \binom{n}{k} = \frac{(r+s)^n}{r}, \tag{T.69}$$

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}, \tag{T.70}$$

$$\sum_{k=0}^{n} [k^3 - (n-k)^3] \binom{n}{k}^3 = 0, \tag{T.71}$$

$$\sum_{k=0}^{n} \binom{m}{k}\binom{n}{k} = \binom{n+m}{m} \quad (\text{Vandermonde}), \tag{T.72}$$

$$\sum_{k=0}^{n} k\binom{n+1}{k}\binom{x}{k} = (n+1)\binom{x+n}{n+1}, \tag{T.73}$$

$$\sum_{k=0}^{n} (-1)^{n+k} \binom{n}{k}\binom{k+b}{k} = \binom{b}{n}, \tag{T.74}$$

$$\sum_{k=0}^{n} (3k-2n)\binom{n}{k}^2\binom{2k}{k} = 0, \tag{T.75}$$

$$\sum_{k=0}^{n} \binom{2n-2k}{n-k}\binom{2k}{k} = 4^n, \tag{T.76}$$

$$\sum_{k=0}^{n} (-1)^k \binom{n+1}{k}\binom{2n-2k+1}{n} = 1, \tag{T.77}$$

$$\sum_{k=0}^{n} 4^{-k}\binom{n}{2k}\binom{2k}{k} = 2^{-(n-1)}\binom{2n-1}{n-1}, \quad (n \geq 1), \tag{T.78}$$

$$\sum_{k=0}^{n} \frac{(-1)^k}{k+1}\binom{n+k}{2k}\binom{2k}{k} = \begin{cases} 1, & \text{falls } n = 0, \\ 0, & \text{falls } n \geq 1, \end{cases} \tag{T.79}$$

$$\sum_{k=0}^{n} \frac{r}{tk+r}\binom{tk+r}{k}\binom{tn-tk+s}{n-k} = \binom{tn+r+s}{n}, \tag{T.80}$$

$$\sum_{k=0}^{n}(-1)^k \binom{2n}{k}\binom{2k}{k}\binom{4n-2k}{2n-k} = \binom{2n}{n}^2, \tag{T.82}$$

$$\sum_{i+j+k=n} \binom{i+j}{i}\binom{j+k}{j}\binom{k+i}{k} = \sum_{0\le j\le n}\binom{2j}{j}, \tag{T.83}$$

$$\sum_{k=0}^{n}(-1)^k \binom{n+b}{n+k}\binom{n+c}{c+k}\binom{b+c}{b+k} = \frac{(n+b+c)!}{n!\,b!\,c!}, \quad \text{(Dixon)} \tag{T.84}$$

$$\exp\left\{\sum_{m\ge 1} m^{m-1}\frac{t^m}{m!}\right\} = 1+\sum_{n\ge 1}(n+1)^{n-1}\frac{t^n}{n!}, \tag{T.85}$$

$$\sum_{k=0}^{2n}(-1)^k \binom{2n}{k}^3 = (-1)^n\frac{(3n)!}{(n!)^3}, \quad \text{(Dixon)} \tag{T.86}$$

$$\sum_{k=0}^{n}(-1)^k \binom{n}{k}\frac{x}{k+x} = \frac{1}{\binom{x+n}{n}}, \tag{T.87}$$

$$\sum_{k=0}^{n}\binom{n}{k}\binom{x}{k+r} = \binom{n+x}{n+r}, \tag{T.88}$$

$$\sum_{k=0}^{n} 2^{2k+1}\binom{x+1}{2k+1}\binom{x-2k}{n-k} = \binom{2x+2}{2n+1}, \tag{T.89}$$

$$(1-2n)\sum_{k=0}^{n}(-4)^k\frac{\binom{n}{k}}{\binom{2k}{k}} = 1, \tag{T.90}$$

$$\sum_{k=0}^{n}(-1)^k\frac{\binom{n}{k}}{\binom{x+k}{k}} = \frac{x}{x+n}, \tag{T.91}$$

$$\sum_{k=0}^{2n} x^k\binom{k}{n} = \left(\frac{x}{1-x}\right)^n\left\{1+x(1-2x)\sum_{j=0}^{n-1}\binom{2j+1}{j}[x(1-x)]^j\right\}. \tag{T.92}$$