

T.6 Transformationen

Es werden mitunter nützliche Substitutionen aufgelistet, wobei

$$W = \begin{vmatrix} \frac{\partial x}{\partial a} & \frac{\partial x}{\partial b} & \frac{\partial x}{\partial c} \\ \frac{\partial y}{\partial a} & \frac{\partial y}{\partial b} & \frac{\partial y}{\partial c} \\ \frac{\partial z}{\partial a} & \frac{\partial z}{\partial b} & \frac{\partial z}{\partial c} \end{vmatrix}$$

die jeweilige WRONSKI-Determinante ist. (Für $W \neq 0$ ist die Transformation eineindeutig.)

T.6.1

$$\left. \begin{array}{l} x = \frac{1}{2}(b+c-a) \\ y = \frac{1}{2}(c+a-b) \\ z = \frac{1}{2}(a+b-c) \end{array} \right\} \iff \begin{cases} a = y+z \\ b = z+x \\ c = x+y, \end{cases}$$

$$x+y+z = \frac{1}{2}(a+b+c), \quad yz+zx+xy = -\frac{1}{4}(a+b+c)^2 + (bc+ca+ab),$$

$$xyz = abc, \quad W = \frac{1}{2}.$$

Dies ist die bekannte Substitution aus Abschnitt G.1.

T.6.2

$$\left. \begin{array}{l} x = (b+c)^k \\ y = (c+a)^k \\ z = (a+b)^k \end{array} \right\} \iff \begin{cases} a = \frac{1}{2}(-x^{\frac{1}{k}} + y^{\frac{1}{k}} + z^{\frac{1}{k}}) \\ b = \frac{1}{2}(x^{\frac{1}{k}} - y^{\frac{1}{k}} + z^{\frac{1}{k}}) \\ c = \frac{1}{2}(x^{\frac{1}{k}} + y^{\frac{1}{k}} - z^{\frac{1}{k}}), \end{cases} \quad k \neq 0,$$

$$W = 2k^3[(b+c)(c+a)(a+b)]^{k-1}.$$

T.6.3

$$\left. \begin{array}{l} x = b+c-ka \\ y = c+a-kb \\ z = a+b-kc \end{array} \right\} \iff \begin{cases} a = \frac{(1-k)x-y-z}{(k+1)(k-2)} \\ b = \frac{(1-k)y-z-x}{(k+1)(k-2)} \\ c = \frac{(1-k)z-x-y}{(k+1)(k-2)} \end{cases} \quad k \neq \{-1, 2\},$$

$$x+y+z = (2-k)(a+b+c),$$

$$yz+zx+xy = (1-2k)(a+b+c)^2 + (k+1)^2(bc+ca+ab),$$

$$xyz = -k(a+b+c)^3 + (k+1)^2(a+b+c)(bc+ca+ab) - (k+1)^3abc,$$

$$W = -(k+1)^2(k-2).$$

T.6.4

$$\left. \begin{array}{l} x = bc \\ y = ca \\ z = ab \end{array} \right\} \iff \begin{cases} a = \sqrt{\frac{yz}{x}} \\ b = \sqrt{\frac{zx}{y}} \\ c = \sqrt{\frac{xy}{z}} \end{cases}$$

T.6.5

$$\left. \begin{array}{l} x = \sqrt{bc} \\ y = \sqrt{ca} \\ z = \sqrt{ab} \end{array} \right\} \iff \left\{ \begin{array}{l} a = \frac{yz}{x} \\ b = \frac{zx}{y} \\ c = \frac{xy}{z} \end{array} \right.$$

$$\begin{aligned} x + y + z &= \sqrt{bc} + \sqrt{ca} + \sqrt{ab}, & yz + zx + xy &= \sqrt{abc}(\sqrt{a} + \sqrt{b} + \sqrt{c}), \\ xyz &= abc, & W &= \frac{1}{4}. \end{aligned}$$