

U.86

$$\frac{bc}{b+c} + \frac{ca}{c+a} + \frac{ab}{a+b} \leq \frac{1}{2}(a+b+c), \quad a, b, c \in \mathbb{R}^+.$$

U.86 Beweis: Aus $(\sqrt{b} - \sqrt{c})^2 \geq 0$ folgt

$$\begin{aligned} 2\sqrt{bc} \leq b+c &\implies \frac{1}{b+c} \leq \frac{1}{2\sqrt{bc}} \implies \frac{bc}{b+c} \leq \frac{\sqrt{bc}}{2} \\ \implies \frac{bc}{b+c} + \frac{ca}{c+a} + \frac{ab}{a+b} &\leq \frac{1}{2}(\sqrt{bc} + \sqrt{ca} + \sqrt{ab}) \leq \frac{1}{2}(a+b+c). \quad \square \end{aligned}$$