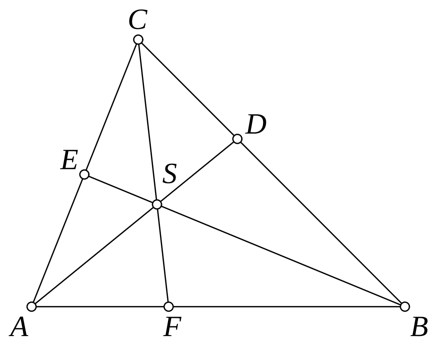
T.1.8 Formeln – Lémoine-Punkt



$$AF = \frac{b^2c}{a^2+b^2}, \qquad BD = \frac{c^2a}{b^2+c^2}, \qquad CE = \frac{a^2b}{c^2+a^2},$$

$$FB = \frac{ca^2}{a^2+b^2}, \qquad DC = \frac{ab^2}{b^2+c^2}, \qquad EA = \frac{bc^2}{c^2+a^2},$$

$$\frac{AF}{FB} = \frac{b^2}{a^2}, \qquad \frac{BD}{DC} = \frac{c^2}{b^2}, \qquad \frac{CE}{EA} = \frac{a^2}{c^2},$$

$$\begin{aligned} AF + BD + CE &= \frac{(a+b+c)a^2b^2c^2 + (ab^3+bc^3+ca^3)abc}{(b^2+c^2)(c^2+a^2)(a^2+b^2)} \\ &\quad + \frac{(b^4c^3+c^4a^3+a^4b^3)+(b^2c^5+c^2a^5+a^2b^5)}{(b^2+c^2)(c^2+a^2)(a^2+b^2)}, \end{aligned}$$

$$\begin{aligned} FB + DC + EA &= \frac{(a+b+c)a^2b^2c^2 + (a^3b+b^3c+c^3a)abc}{(b^2+c^2)(c^2+a^2)(a^2+b^2)} \\ &\quad + \frac{(b^3c^4+c^3a^4+a^3b^4)+(b^5c^2+c^5a^2+a^5b^2)}{(b^2+c^2)(c^2+a^2)(a^2+b^2)}, \end{aligned}$$

$$AD = \frac{bc}{b^2+c^2}\sqrt{2(b^2+c^2)-a^2}, \qquad BE = \frac{ca}{c^2+a^2}\sqrt{2(c^2+a^2)-b^2},$$

$$CF = \frac{ab}{a^2+b^2}\sqrt{2(a^2+b^2)-c^2},$$

$$\frac{AS}{SD} = \frac{b^2+c^2}{a^2}, \qquad \frac{SD}{AD} = \frac{a^2}{a^2+b^2+c^2}, \qquad \frac{AS}{AD} = \frac{b^2+c^2}{a^2+b^2+c^2},$$

$$\frac{BS}{SE} = \frac{c^2+a^2}{b^2}, \qquad \frac{SE}{BE} = \frac{b^2}{a^2+b^2+c^2}, \qquad \frac{BS}{BE} = \frac{c^2+a^2}{a^2+b^2+c^2},$$

$$\frac{CS}{SF} = \frac{a^2+b^2}{c^2}, \qquad \frac{SF}{CF} = \frac{c^2}{a^2+b^2+c^2}, \qquad \frac{CS}{CF} = \frac{a^2+b^2}{a^2+b^2+c^2},$$

$$AS = \frac{bc}{a^2+b^2+c^2}\sqrt{2(b^2+c^2)-a^2}, \qquad BS = \frac{ca}{a^2+b^2+c^2}\sqrt{2(c^2+a^2)-b^2},$$

$$CS = \frac{ab}{a^2+b^2+c^2}\sqrt{2(a^2+b^2)-c^2},$$

$$SD = \frac{a^2bc}{(a^2+b^2+c^2)(b^2+c^2)}\sqrt{2(b^2+c^2)-a^2},$$

$$SE = \frac{ab^2c}{(a^2+b^2+c^2)(c^2+a^2)}\sqrt{2(c^2+a^2)-b^2},$$

$$SF = \frac{abc^2}{(a^2+b^2+c^2)(a^2+b^2)}\sqrt{2(a^2+b^2)-c^2},$$

$$\sin\measuredangle ACF = \frac{c\sin\beta}{\sqrt{2(a^2+b^2)-c^2}}, \qquad \sin\measuredangle BCF = \frac{c\sin\alpha}{\sqrt{2(a^2+b^2)-c^2}},$$

$$\sin\measuredangle BAD = \frac{a\sin\gamma}{\sqrt{2(b^2+c^2)-a^2}}, \qquad \sin\measuredangle CAD = \frac{a\sin\beta}{\sqrt{2(b^2+c^2)-a^2}},$$

$$\sin\measuredangle CBE = \frac{b\sin\alpha}{\sqrt{2(c^2+a^2)-b^2}}, \qquad \sin\measuredangle ABE = \frac{b\sin\gamma}{\sqrt{2(c^2+a^2)-b^2}}.$$

(Wird fortgesetzt.)