T.6 Transformationen

Es werden mitunter nützliche Substitutionen aufgelistet, wobei

$$W = \begin{vmatrix} \frac{\partial x}{\partial a} & \frac{\partial x}{\partial b} & \frac{\partial x}{\partial c} \\ \frac{\partial y}{\partial a} & \frac{\partial y}{\partial b} & \frac{\partial y}{\partial c} \\ \frac{\partial z}{\partial a} & \frac{\partial z}{\partial b} & \frac{\partial z}{\partial c} \end{vmatrix}$$

die jeweilige Wronski-Determinante ist. (Für $W \neq 0$ ist die Transformation eineindeutig.)

T.6.1

$$\left.\begin{aligned} x &= \tfrac{1}{2}(b+c-a) \\ y &= \tfrac{1}{2}(c+a-b) \\ z &= \tfrac{1}{2}(a+b-c) \end{aligned}\right\} \iff \left\{\begin{aligned} a &= y+z \\ b &= z+x \\ c &= x+y, \end{aligned}\right.$$

$$x+y+z = \frac{1}{2}(a+b+c), \qquad yz+zx+xy = -\frac{1}{4}(a+b+c)^2 + (bc+ca+ab),$$

$$xyz = abc, \qquad W = \frac{1}{2}.$$

Dies ist die bekannte Substitution aus Abschnitt G.1.

T.6.2

$$\left.\begin{aligned} x &= (b+c)^k \\ y &= (c+a)^k \\ z &= (a+b)^k \end{aligned}\right\} \iff \left\{\begin{aligned} a &= \tfrac{1}{2}\left(-x^{\frac{1}{k}} + y^{\frac{1}{k}} + z^{\frac{1}{k}}\right) \\ b &= \tfrac{1}{2}\left(x^{\frac{1}{k}} - y^{\frac{1}{k}} + z^{\frac{1}{k}}\right) \\ c &= \tfrac{1}{2}\left(x^{\frac{1}{k}} + y^{\frac{1}{k}} - z^{\frac{1}{k}}\right), \end{aligned}\right. \qquad k \neq 0,$$

$$W = 2k^3[(b+c)(c+a)(a+b)]^{k-1}.$$

T.6.3

$$\left.\begin{aligned} x &= b+c-ka \\ y &= c+a-kb \\ z &= a+b-kc \end{aligned}\right\} \iff \left\{\begin{aligned} a &= \tfrac{(1-k)x-y-z}{(k+1)(k-2)} \\ b &= \tfrac{(1-k)y-z-x}{(k+1)(k-2)} \\ c &= \tfrac{(1-k)z-x-y}{(k+1)(k-2)} \end{aligned}\right. \qquad k \neq \{-1, 2\},$$

$$x+y+z = (2-k)(a+b+c),$$

$$yz+zx+xy = (1-2k)(a+b+c)^2 + (k+1)^2(bc+ca+ab),$$

$$xyz = -k(a+b+c)^3 + (k+1)^2(a+b+c)(bc+ca+ab) - (k+1)^3abc,$$

$$W = -(k+1)^2(k-2).$$

T.6.4

$$\left.\begin{aligned} x &= bc \\ y &= ca \\ z &= ab \end{aligned}\right\} \iff \left\{\begin{aligned} a &= \sqrt{\tfrac{yz}{x}} \\ b &= \sqrt{\tfrac{zx}{y}} \\ c &= \sqrt{\tfrac{xy}{z}} \end{aligned}\right.$$

$$x+y+z = bc+ca+ab, \qquad yz+zx+xy = abc(a+b+c),$$

$$xyz = (abc)^2, \qquad W = 2abc.$$

T.6.5

$$\left.\begin{aligned} x &= \sqrt{bc} \\ y &= \sqrt{ca} \\ z &= \sqrt{ab} \end{aligned}\right\} \quad \Longleftrightarrow \quad \left\{\begin{aligned} a &= \tfrac{yz}{x} \\ b &= \tfrac{zx}{y} \\ c &= \tfrac{xy}{z} \end{aligned}\right.$$

$$x + y + z = \sqrt{bc} + \sqrt{ca} + \sqrt{ab}, \qquad yz + zx + xy = \sqrt{abc}\,(\sqrt{a} + \sqrt{b} + \sqrt{c}),$$

$$xyz = abc, \qquad W = \frac{1}{4}.$$