U.73 Es gelten folgende Ungleichungen in den elementaren symmetrischen Funktionen $\sigma_1 \equiv a+b+c$, $\sigma_2 \equiv bc+ca+ab$ und $\sigma_3 \equiv abc$:

Polynome 2. Grades

$$\sigma_1^2 \geq 3\sigma_2, \tag{U.64}$$

Polynome 3. Grades

$$\sigma_1^3 \geq 27\sigma_3, \tag{U.65}$$

$$\sigma_1\sigma_2 \geq 9\sigma_3, \tag{U.66}$$

$$2\sigma_1^3 + 9\sigma_3 \geq 7\sigma_1\sigma_2, \tag{U.67}$$

$$\sigma_1^3 + 9\sigma_3 \geq 4\sigma_1\sigma_2, \tag{U.68}$$

Polynome 4. Grades

$$\sigma_2^2 \geq 3\sigma_1\sigma_3, \tag{U.69}$$

$$\sigma_1(\sigma_1\sigma_2 + 3\sigma_3) \geq 4\sigma_2^2, \tag{U.70}$$

$$\sigma_1^2\sigma_2 \geq 3\sigma_1\sigma_3 + 2\sigma_2^2, \tag{U.71}$$

$$\sigma_1^4 + 3\sigma_1\sigma_3 + 5\sigma_2^2 \geq 5\sigma_1^2\sigma_2, \tag{U.72}$$

$$\sigma_1^4 + 6\sigma_1\sigma_3 + 4\sigma_2^2 \geq 5\sigma_1^2\sigma_2, \tag{U.73}$$

$$\sigma_1^4 + 3\sigma_2^2 \geq 4\sigma_1^2\sigma_2, \tag{U.74}$$

Polynome 5. Grades

$$\sigma_2(\sigma_1^3 + 27\sigma_3) \geq 3\sigma_1(3\sigma_1\sigma_3 + \sigma_2^2), \tag{U.75}$$

$$\sigma_2(\sigma_1\sigma_2 + 3\sigma_3) \geq 4\sigma_1^2\sigma_3, \tag{U.76}$$

$$\sigma_1(2\sigma_1^4 + 9\sigma_1\sigma_3 + 21\sigma_2^2) \geq \sigma_2(13\sigma_1^3 + 27\sigma_3), \tag{U.77}$$

$$\sigma_1\sigma_2^2 \geq \sigma_3(2\sigma_1^2 + 3\sigma_2), \tag{U.78}$$

$$\sigma_1^3\sigma_2 + \sigma_1^2\sigma_3 + 6\sigma_2\sigma_3 \geq 4\sigma_1\sigma_2^2, \tag{U.79}$$

$$\sigma_2(\sigma_1^3 + 15\sigma_3) \geq \sigma_1(5\sigma_1\sigma_3 + 3\sigma_2^2), \tag{U.80}$$

$$\sigma_1(2\sigma_1^4 + 10\sigma_1\sigma_3 + 17\sigma_2^2) \geq 3\sigma_2(4\sigma_1^3 + 7\sigma_3), \tag{U.81}$$

$$\sigma_1(2\sigma_1^4 + 11\sigma_1\sigma_3 + 13\sigma_2^2) \geq \sigma_2(11\sigma_1^3 + 15\sigma_3), \tag{U.82}$$

$$\sigma_1(\sigma_1^4 + 9\sigma_1\sigma_3 + 12\sigma_2^2) \geq \sigma_2(7\sigma_1^3 + 27\sigma_3), \tag{U.83}$$

$$\sigma_1(\sigma_1^4 + 7\sigma_1\sigma_3 + 8\sigma_2^2) \geq 6\sigma_2(\sigma_1^3 + 2\sigma_3), \tag{U.84}$$

$$\sigma_1(\sigma_1^4 + 7\sigma_1\sigma_3 + 4\sigma_2^2) \geq \sigma_2(5\sigma_1^3 + 3\sigma_3), \tag{U.85}$$

Polynome 6. Grades

$$\sigma_1(2\sigma_1^5 + 57\sigma_1\sigma_2^2 + 54\sigma_2\sigma_3) \geq 3(6\sigma_1^4\sigma_2 + 4\sigma_1^3\sigma_3 + 22\sigma_2^3 + 27\sigma_3^2), \tag{U.86}$$

$$\sigma_1^2\sigma_2^2 + 27\sigma_3^2 \geq 2(\sigma_1^3\sigma_3 + \sigma_2^3), \tag{U.87}$$

$$\sigma_1^4\sigma_2 + 7\sigma_1^3\sigma_3 + 16\sigma_2^3 + 27\sigma_3^2 \geq \sigma_1\sigma_2(8\sigma_1\sigma_2 + 27\sigma_3), \tag{U.88}$$

$$\sigma_2(\sigma_1^4 + 27\sigma_1\sigma_3 + 2\sigma_2^2) \geq 7\sigma_1^3\sigma_3 + 4\sigma_1^2\sigma_2^2 + 27\sigma_3^2, \tag{U.89}$$

$$\sigma_1\sigma_2(\sigma_1\sigma_2 + 10\sigma_3) \geq 4\sigma_1^3\sigma_3 + 2\sigma_2^3 + 9\sigma_3^2, \tag{U.90}$$

$$2\sigma_2^3 + 9\sigma_3^2 \geq 7\sigma_1\sigma_2\sigma_3, \tag{U.91}$$

$$\sigma_1^2(2\sigma_1^4 + 2\sigma_1\sigma_3 + 41\sigma_2^2) \geq 16\sigma_1^4\sigma_2 + 34\sigma_2^3 + 27\sigma_3^2, \tag{U.92}$$

$$2\sigma_1^6 + 12\sigma_1^3\sigma_3 + 51\sigma_1^2\sigma_2^2 + 81\sigma_3^2 \geq 6\sigma_2(3\sigma_1^4 + 9\sigma_1\sigma_3 + 7\sigma_2^2), \tag{U.93}$$

$$\sigma_1\sigma_2(\sigma_1\sigma_2 + 18\sigma_3) \geq 4\sigma_1^3\sigma_3 + 4\sigma_2^3 + 27\sigma_3^2, \tag{U.94}$$

$$\sigma_1^6 + 10\sigma_1^3\sigma_3 + 18\sigma_1^2\sigma_2^2 + 27\sigma_3^2 \geq 4\sigma_2(2\sigma_1^4 + 9\sigma_1\sigma_3 + 2\sigma_2^2), \tag{U.95}$$

$$3\sigma_1\sigma_2(\sigma_1\sigma_2 + 12\sigma_3) \geq 10\sigma_1^3\sigma_3 + 10\sigma_2^3 + 27\sigma_3^2, \tag{U.96}$$

$$\sigma_1^6 + 8\sigma_1^3\sigma_3 + 13\sigma_1^2\sigma_2^2 + 9\sigma_3^2 \geq \sigma_2(7\sigma_1^4 + 22\sigma_1\sigma_3 + 4\sigma_2^2), \tag{U.97}$$

$$\sigma_2^3 + 9\sigma_3^2 \geq 4\sigma_1\sigma_2\sigma_3, \tag{U.98}$$

$$\sigma_1^6 + 8\sigma_1^3\sigma_3 + 8\sigma_1^2\sigma_2^2 + 9\sigma_3^2 \geq 2\sigma_1\sigma_2(3\sigma_1^3 + 8\sigma_3), \tag{U.99}$$

$$\sigma_1^3\sigma_3 + \sigma_2^3 + 9\sigma_3^2 \geq 7\sigma_1\sigma_2\sigma_3, \tag{U.100}$$

$$\sigma_1^2(\sigma_1^4 + 6\sigma_1\sigma_3 + 9\sigma_2^2) \geq 3\sigma_2(2\sigma_1^4 + 3\sigma_1\sigma_3 + \sigma_2^2), \tag{U.101}$$

$$\sigma_1^2(\sigma_1^4 + 4\sigma_1\sigma_3 + 9\sigma_2^2) \geq 2\sigma_2(3\sigma_1^4 + 3\sigma_1\sigma_3 + \sigma_2^2), \tag{U.102}$$

$$\sigma_1^3\sigma_3 + \sigma_2^3 \geq 6\sigma_1\sigma_2\sigma_3. \tag{U.103}$$

U.73 Die angegebenen Ungleichungen folgen durch „Größer-Gleich-null-Setzen“ der in Tabelle T.4 aufgeführten nichtnegativen Ausdrücke. (Eine Ausnahme ist (U.64), die durch Multiplikation von (U.63) mit (U.65) und anschließender Division durch σ_2 folgt.)